A decomposition-based approach for large-scale pickup and delivery problems

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Abstract

With the advent of self-driving cars, experts envision autonomous mobility-on-demand services in the near future to cope with overloaded transportation systems in cities worldwide. Efficient operations are imperative to unlock such a system's maximum improvement potential. Existing approaches either consider a narrow planning horizon or ignore essential characteristics of the underlying problem. In this paper, we develop an algorithmic framework that allows the study of very large-scale pickup and delivery routing problems with more than 20 thousand requests, which arise in the context of integrated request pooling and vehicle-to-request dispatching. We conduct a computational study and present comparative results showing the characteristics of the developed approaches. Furthermore, we apply our algorithm to related benchmark instances from the literature to show the efficacy. Finally, we solve very large-scale instances and derive insights on upper-bound improvements regarding fleet sizing and customer delay acceptance from a practical perspective.

Keywords: large scale problems; pickup and delivery; ride-sharing; optimization

1. Introduction

Cities worldwide struggle with overloaded transportation systems and related negative externalities: CO₂ emissions cause environmental harm by contributing to the greenhouse effect; health hazards arise from particulate matter and NO_x emissions; and economic harm stems from congestion-induced lost working hours (Pishue 2023). Over the last decade, the sharing economy paradigm stimulated new mobility services, above all mobility-on-demand (MoD) services to serve individual passenger transportation requests in an urban context. With the advent of self-driving cars, experts envision these MoD services to be operated by autonomous vehicles in the near future. The resulting autonomous MoD (MoD) systems promise to enable more efficient on-demand services that are accessible by a large public as they can be offered at a lower cost. Municipalities, practitioners, and scientists share high hopes that such AMoD systems will contribute significantly to reducing the above mentioned negative externalities by allowing, among others, for efficient pooling of passenger requests, congestion aware routing, and convenient feeding to public transport lines (Salazar et al. 2019). Yet, there is so far no consensus whether these benefits will finally lead to reduced negative externalities, or if such a convenient on-demand service may lead to induced demand, such that externalities will be reduced per passenger but not in total (Oh et al. 2020).

Independent of this debate, there is consensus that efficient operations are imperative to unlock an AMoD system's maximum improvement potential. To do so, one needs to develop effective, possibly anticipative, control algorithms for the related operational planning tasks: pooling–if possible– passenger requests to shared rides, dispatching these rides to vehicles, and rebalancing vehicles to anticipate future demand. Accordingly, these planning problems sparked the interest of scientists in the field of optimal control, transport optimization, and operations research and led to various algorithmic approaches that range from online control algorithms, often following a (partial) planning-task decomposition scheme (Enders et al. 2023, 2024), to integrated algorithms that aim to solve all planning tasks at once in a full information setting to identify an upper bound on the possible system improvement.

In this context, existing approaches usually suffer from at least one shortcoming: (decomposed) online control algorithms scale to large real-world instances but implicitly assume the performance loss of sequentially deciding on the respective planning tasks to be limited without further discussion (Alonso-Mora et al. 2017). Consequently, system improvement analyses are limited to comparisons against the status quo or (naive) baselines. While this setting reveals many interesting insights, it is not sufficient to rigorously analyze the upper bounds of a system's improvement potential, which is of interest for tactical and strategic transportation system analysis and planning. Algorithms that solve the integrated planning problem in a full information setting are often limited in scalability (Doerner & Salazar-González 2014, Sartori & Buriol 2020), such that the obtained full information bounds are not interesting from a practitioner's perspective as the limited problem size diminishes the meaningfulness of the derived results. In the field of operations research, some first attempts exist to solve large-scale pickup and delivery problems in a full information setting. However, these works are motivated by freight transport applications, such that the studied instance size

still remains below the scale encountered in an AMoD context. Moreover, the instances studied in this work reveal significantly different characteristics with respect to demand and time windows such that it remains questionable whether existing–usually highly-tailored–algorithms will provide a good solution quality within an AMoD context.

Against this background, we aim at developing an algorithmic framework that allows to solve very large-scale pickup and delivery routing problems that arise in the context of integrated request pooling and vehicle-to-request dispatching in AMoD systems. Focusing on offline full information problem settings, we design this algorithmic framework in such a way that it allows us to analyze not only a full information bound of the system's performance improvement but also the impact of taking pooling and dispatching decisions sequentially or in an integrated fashion.

In the following, we first briefly review related literature before we specify our contribution and the paper's organization.

1.1. State of the Art

From a general perspective, controlling a ride-hailing fleet is related to solving an pickup and delivery problem with time windows (PDPTW) (Savelsbergh & Sol 1995, Ropke et al. 2007). Literature on the PDPTW focuses overwhelmingly on deliveries of goods (Battarra et al. 2014). Apart from its basic problem variant, several extensions for the PDPTW have been studied, e.g., focusing on loading constraints (Iori & Martello 2010), selection of requests (Al-Chami et al. 2016), or recharging of electric vehicles (Goeke 2019). Passenger transportation variants are studied under the umbrella of the dial-a-ride problem (DARP), which was conceived to formalize a mobility service for elderly and disabled people to maximize the quality of service while minimizing cost. Herein, not only punctuality in service but the time passengers travel in the vehicle are considered using the aptly called user-ride-time constraint to limit the ride duration and avoid long periods of waiting times in the vehicle (Doerner & Salazar-González 2014). This work focuses on inner-city ride-sharing operations with tight time windows regarding pickup and dropoff and short trips. As such, the maximum ride times of users are bounded by the time windows, removing the necessity of using explicit limits. For this reason, we consider the PDPTW as our modeling basis and focus our literature review on this problem.

Only a few exact approaches exist to solve the PDPTW. Besides some early branch-and-price solutions (Dumas et al. 1991, Savelsbergh & Sol 1998), branch-and-price-and-cut algorithms have been developed by Ropke & Cordeau (2009) and Baldacci et al. (2011) with Bettinelli et al. (2014) and Gschwind et al. (2018) focusing on how to apply bidirectional search in the pricing problem. The largest instances that could be solved with these approaches comprise 100 requests for regular instances and up to 500 requests for tightly constrained instances. Recently, Vadseth et al. (2023) proposed a route modifying improvement model, which they initialized with best-known solutions to find improvements for the large benchmark sets.

To solve larger instances, different metaheuristic resolution approaches have been developed, covering Tabu Search (Nanry & J.W. 2000, Li & Lim 2001), genetic algorithm Pankratz (2005)

and hyper-heuristics (Nasiri et al. 2022). Nagata & Kobayashi (2010) proposed a guided ejection search (GES) and focused on minimizing the fleet only. Current state-of-the-art methods include an iterative approach by Curtois et al. (2018), using an adaptive guided ejection search (AGES), large neighborhood search (LNS) and local search (LS) sequentially to repeatedly probe minimizing the fleet before optimizing costs. Sartori & Buriol (2020) later improved upon this approach and proposed a matheuristic composing of a AGES, LNS, and an set partitioning (SP) component. The latter continuously tries to recombine routes of solutions found during the search to yield better solutions. These components are nested in an iterated local search (ILS) framework to diversify the search. Christiaens & Vanden Berghe (2020) proposed an ruin & recreate (R&R) using a novel slackinduced string removal and greedy insertion with blinks operator. The approach showed excellent performance on various problems, including the PDPTW.

In contrast with the literature, our focus lies on very large instances, ranging up to 21 thousand requests compared to the currently studied 2500 available requests.

1.2. Contribution

With this work, we provide a new state of the art for solving very large scale PDPTWs in the context of ride-sharing. We provide an algorithmic framework that comprises a decomposition-based matheuristic which allows to solve instances with up to 5000 requests in a few minutes, an ILS-based metaheuristic that yields a better solution quality within a computational time limit of up to 15 minutes, as well as a hybrid approach that uses our matheuristic to warm-start the ILS in order to further improve solution quality.

With this algorithmic framework, we provide a thorough computational study to compare the proposed algorithms against each other and understand the respective algorithmic characteristics. We then use our metaheuristic algorithm to solve the benchmark data set for the PDPTW in the context of ridesharing Sartori & Buriol (2020). Here, we show that our algorithm improves significantly over the algorithm of Sartori & Buriol (2020) and find new best-known solutions. Lastly, we apply our algorithm to study very large-scale instances with up to 21375 requests that have not been solved before. By so doing, we shed light on upper-bound improvements with respect to fleet sizing and customer delay acceptance from a practical perspective.

1.3. Organization

The remainder of this paper is organized as follows: Section 2 formally introduces our problem setting. Section 3 details the proposed solution approach. In Section 4, we outline our experimental design and present the related numerical results in Section 5. Finally, Section 7 concludes this work with a short summary and an outlook on future research.

2. Problem Setting

We study an offline problem setting in which a fleet operator has full knowledge about requests that arrive in a certain time horizon, e.g., a peak hour, an operational shift, or a day. Formally, we denote this time horizon by \mathcal{T} . The fleet operator operates a fleet of constant size and offers a ride-hailing service. In this context the operator decides on i) which customer requests to pool to a shared ride, and ii) which vehicle to dispatch for operating each (shared) ride. Clearly, the operator can also decide to not dispatch a vehicle to a ride, which implicitly models request rejections.

The fleet operates on a road network and we model respective operations on a fully connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ composed of a set of nodes $\mathcal{N} = P \cup \mathcal{D} \cup \mathcal{K}$ and a set of arcs \mathcal{A} . Here, Set P contains the pickup nodes of all passenger requests, \mathcal{D} contains the respective drop-off nodes, and set \mathcal{K} contains vehicle nodes, i.e., nodes that indicate the initial positions of all vehicles. Arcs $(i, j) \in \mathcal{A}$ represent paths through the street network with corresponding cost c_{ij} and travel time t_{ij} .

We represent a passenger request as a quadruple (p_r, d_r, e_r, l_r) , where $p_r \in P$ and $d_r \in \mathcal{D}$ denote the pickup and dropoff location of request $r \in R$, and e_r, l_r define the request's time window $[e_r, l_r]$. This time window indicates the earliest time $e_r \in \mathcal{T}$ at which it is possible to pick up the passenger at p_r and the latest time $l_r \in \mathcal{T}$ at which the passenger needs to be dropped at d_r . In this context, one can interpret e_r either as the time at which a passenger sends a request to the operator, or as a specified pickup time that lies further in the future. To account for a passengers willingness to accept a detour when participating in a shared ride, e.g., incentivized by a price reduction, we calculate l_r as

$$l_r = e_r + t_{p_r d_r} + \delta_r$$

where $t_{p_rd_r}$ is the travel time between p_r and d_r , and δ denotes a maximum time budget available for detours. In the remainder of this paper, we will refer to this time budget as a request's buffer.

With this notation, one can easily link a request's time window information to its origin and destination. We do so by defining a pickup time window $[e_{p_r}, l_{p_r}]$ as $e_{p_r} = e_r$ and $l_{p_r} = e_r + \delta$, and a delivery time window $[e_{d_r}, l_{d_r}]$ as $e_{d_r} = e_r + t_{p_r d_r}$ and $l_{d_r} = l_r = e_r + t_{p_r d_r} + \delta$ for each request. While these definitions appear to be redundant in the problem description, they will ease notation and clarity when discussing our algorithmic framework.

Solution representation: A solution σ represents a set of routes $\sigma = \{\vartheta^1, \ldots, \vartheta^{|\mathcal{K}|}\}$, one for each vehicle. Each route $\vartheta^k = (\nu_0, \ldots, \nu_n)$ is a sequence of nodes $\nu_i \in \mathcal{N}$ that denotes in which order a vehicle visits them. Here, ν_0 is always the starting location of the corresponding vehicle. Each sequence implicitly defines departure and arrival times, which can be trivially calculated by propagating travel times, departing as early as possible from each node. We define $\tau^{\text{ARR}}(\vartheta, \nu_i)$ and $\tau^{\text{DEP}}(\vartheta, \nu_i)$ to access the arrival and departure time of node $\nu_i \in \mathcal{N}$. Finally, we define $\mathcal{U}(\sigma)$ as the set of unassigned requests that are not served by any vehicle within a solution σ .

Constraints: A valid solution σ has to adhere to the following constraints.

- i Vehicles can never exceed their capacity Q and can thus operate at maximum Q requests in parallel.
- ii If served, request nodes p_r and d_r have to be visited by the same vehicle, i.e.,

$$p_r \in \vartheta \leftrightarrow d_r \in \vartheta, \quad \forall r \in R$$
 (2.1)

iii Request nodes must be served in order and inside the corresponding requests' time window $[e_r, l_r]$. If a vehicle arrives early, it has to wait until the request can be served. Formally

$$e_r \le \tau^{\text{DEP}}(p_r) \le \tau^{\text{ARR}}(d_r) \le l_r, \quad \forall r \in R,$$

$$(2.2)$$

Objective function: We consider a hierarchical objective, minimizing two quantities: i) the number of unserved requests in $\mathcal{U}(\sigma)$, and ii) the total travel cost considering the driving distance c_{ij} .

min
$$|\mathcal{U}(\sigma)|$$

min $\sum_{\vartheta \in \sigma} \sum_{i=1}^{|\vartheta|} c_{\nu_{i-1}\nu_i}$ (2.3)

Among all feasible solutions fulfilling these constraints, we seek a solution σ^* that minimizes the objective function (2.3).

Discussion: Three comments on our problem setting are in order. First, we limit our problem setting to a full information scenario, which omits directly leveraging our algorithm for fleet control in practice. While limiting, this simplification is in line with our paper's scope: solving large-scale full information instances to obtain upper bounds on the system's improvement potential, as well as a temporally unbiased analyses on whether decomposing pooling and dispatching decisions affects solution quality or not. Second, while we analyze the impact of decoupled or integrated pooling and dispatching decisions, we ignore explicit rebalancing and rerouting. Omitting rebalancing is reasonable as it is only beneficial during online decision making but not in a full information setting. For (congestion aware) rerouting, recent works show significant improvement potential even if it is conducted on a subsequent decision level (Jalota et al 2023). Accordingly, we ignore this aspect to isolate the effect of decomposing or integrating pooling and dispatching decisions. Lastly, we like to mention that the proposed problem setting differs from classical PDPTWs in three aspects: i) we do not consider heterogeneous demand, instead each request has a demand of one, i.e., represents one customer; ii) our setting resembles an orienteering problem where vehicles are initially scattered within the service area instead of being located at a central depot and may also end their last service at an arbitrary location; iii) we consider a different objective function that aims at minimizing the number of unserved requests and subsequently the respective operational costs for a given fleet size. All of these differences result from our ride-hailing application, which significantly differs from the usually studied logistics context.

3. Methodology

This section details our algorithmic framework. To obtain a framework that allows to take decomposed as well as integrated pooling and dispatching decisions, we proceed as follows. In a first step, we develop a matheuristic that decomposes the planning problem and takes pooling and dispatching decisions sequentially. We then focus on integrated decision-making and develop a metaheuristic that allows to take integrated pooling and dispatching decisions.

3.1. Sequential Pooling & Dispatching

To devise an algorithm for sequential pooling and dispatching decisions, we expand two of our recent works that focused on the respective isolated decision tasks. Figure 1 illustrates the rationale of our algorithm. In a first step (Figure 1a), we model potential request poolings as hyperedges in a hypergraph, which allows us to effectively pool customer requests by obtaining a maximum weighted matching. We then model these pooled requests in a dispatching graph (Figure 1b), which allows us to calculate the request to vehicle assignments in polynomial time.

3.1.1. Generating feasible hyperedges

We define a hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$ with a vertex set \mathcal{V} , a hyperedge set \mathcal{E} , and its hyperedge weights ω , which relates to our problem as follows:

- i The vertex set is ordered $\mathcal{V} = [n]$, (with $n = |\mathcal{V}|$) and each vertex represents a request $r \in R$. The set is sorted by the earliest service time e_r in ascending order.
- ii Each hyperedge ε with $|\varepsilon| \ge 2$ represents a potential pooled ride that contains multiple requests.
- iii The edge weights $\omega : \mathcal{E} \to \mathbb{R}_+$, represent the utility of a potentially pooled ride.

The number of hyperedges can be potentially intractable, with up to $\sum_{\gamma=2}^{|R|} {|R| \choose \gamma}$ possible combinations. To tackle this issue, we limit the hyperedges generated in two ways. First, we limit the rank of the hypergraph to $\gamma^{\text{MAX}} = 4$, i.e., $|\varepsilon| \leq \gamma^{\text{MAX}}, \forall \varepsilon \in \mathcal{E}$. Second, we define N_r as the set of feasible neighbors of request r, where $e_r \leq e_{r'} \leq l_r + \delta$. For each request r and its neighbors N_r , we



Figure 1: Outline of the sequential pooling and dispatching approach.

(a) Step 1: Hypergraph matching.

(b) Step 2: Create and solve a dispatching graph.

generate every combination $\pi \in \Pi_r$ of up to γ^{MAX} requests, $\pi \in \{r\} \cup N_r$. We only allow $r' \in N_r$ if r' is positioned after r in the ordered vertex set \mathcal{V} to avoid symmetries.

Note that not all possible hyperedge combinations may result in a feasible sequence as they may violate the pickup time feasibility or capacity constraint, and can be ignored. Furthermore, a set of requests can be sequenced in various ways while respecting the precedence constraint. For example, $\pi = \{r, r'\}$ can be sequenced in four ways: $(p_r, p_{r'}, d_r, d_{r'}), (p_r, p_{r'}, d_{r'}, d_r), (p_{r'}, p_r, d_r, d_{r'}), (p_{r'}, p_r, d_r, d_r), (p_{r'}, p_r, d_r)$

3.1.2. Hypergraph matching

We define a matching \mathcal{M} for hypergraph \mathcal{H} as a subset of hyperedges $\mathcal{M} \subset \mathcal{E}$ where all edges are disjoint. Such a matching is maximal if it is not a strict subset of any other matching, and it is maximum if no other matching with a greater cardinality exists. Then, the best pooling strategy \mathcal{M}^* equals a maximum weight matching in \mathcal{H} , formally

$$\mathcal{M}^* := \arg \max_{X \in \mathcal{E}} \sum_{\varepsilon \in \mathcal{E}} \omega(\varepsilon)$$

s.t. $\varepsilon_i \cup \varepsilon_j = \emptyset, \forall \varepsilon_i, \varepsilon_j \in \mathcal{E}$

Clearly, the definition of the hyperedge weights ω is crucial for the actual matching performance. In this work, we analyze four weight functions to evaluate hyperedges $\varepsilon \in \mathcal{E}$ as follows.

$$\omega^{1}(\varepsilon) = -\gamma^{\text{MAX}} / \gamma_{\varepsilon} \tag{3.1}$$

$$\omega^2(\varepsilon) = \left(\sum_{(i,j)\in\varepsilon} c_{ij}\right) \cdot \omega^1 \tag{3.2}$$

$$\omega^{3}(\varepsilon) = \left(\left(\max_{r\in\varepsilon}\{l_{r}\} - \min_{r\in\varepsilon}\{e_{r}\}\right) - \left(\min_{r\in\varepsilon}\{l_{r}\} - \max_{r\in\varepsilon}\{e_{r}\}\right)\right) \cdot \omega^{1}$$
(3.3)

$$\omega^4(\varepsilon) = \omega^2(1-\rho) + \omega^3\rho \tag{3.4}$$

Here, $\omega^1(\varepsilon)$ parameterizes each edge based on the negative inverse of its cardinality normalized by the maximum cardinality of hyperedge ε . While this negative inverse seems unintuitive at first sight, it appears reasonable once we embedd the respective poolings in our dispatching algorithm, which bases on solving a k-disjoint shortest path problem (k-dSPP) on a dispatching graph. To do so, it requires a negative representation of each poolings benefit. We reach such a notion with respect to each poolings utilization by considering the negative inverse of a utilization ratio expressed via the hyperedge cardinality. In the remaining weight definitions, we use ω_1 as a normalization factor such that no further transformations are necessary to define the other weights in a metric that is suitable for the subsequent dispatching algorithm. The remaining weights aim to either account for the cost of the resepctive request sequence (ω^2), the temporal overlap between the pooled requests (ω^3), or a convex combination of both (ω^4). To find \mathcal{M} , we use a two-step matheuristic approach. First, we solve a continuous weighted set covering (WSC) problem to select a subset of hyperedges that may be connected to the same request nodes, i.e., a fractional matching $\overline{\mathcal{M}}$. We then derive an integral matching \mathcal{M} based on the solution of the WSC using a greedy selection procedure.

We define our continuous WSC as follows.

$$\max \quad \sum_{\varepsilon \in \mathcal{E}'} \omega(\varepsilon) \cdot x_{\varepsilon} \tag{3.5}$$

$$\sum_{\varepsilon \in \mathcal{E}'} a_{r\varepsilon} x_{\varepsilon} \ge 1 \qquad \qquad \forall r \in R \qquad (3.6)$$

$$0 \le x_{\varepsilon} \le 1 \qquad \qquad \forall \varepsilon \in \mathcal{E}' \tag{3.7}$$

Here, $a_{r\varepsilon} = 1$ if request r is connected to hyperedge ε and remains zero otherwise; while $\mathcal{E}' = \mathcal{E} \cup \bigcup_{r \in R} \varepsilon_r$ is the set of hyperedges including hyperedges comprising only a single request $r \in R$.

After solving the continuous WSC problem, we obtain matching \mathcal{M} from the fractional matching $\overline{\mathcal{M}}$ using a greedy selection procedure. Herein, we traverse the hyperedges $\varepsilon \in \overline{\mathcal{M}}$, sorted by their fractional solution value in descending order, to construct a matching \mathcal{M} . We only add a hyperedge to our matching if it does not contain any request already covered in \mathcal{M} . The process stops after all requests have been selected or the list has been traversed.

We tested additional approaches to find good matchings in preliminary experiments, including a simple greedy selection approach based on the hyperedge weight $\omega(\varepsilon)$, and a weighted set partitioning reformulation (see Appendix A). Although the aggregated results do slightly favor the greedy approach, the disaggregated results on fleet sizes relevant in practice show better performance when using the matheuristic approach with ω^4 and $\rho = 0.7$.

3.1.3. Dispatching

Preliminarily analyses of the characteristics of our planning problem showed that requests in our ride hailing content often tend to interlace less compared to classical DARP instances. Accordingly, we often observe subsequences of visits that start and end with an empty vehicle, often referred to as zero-split sequences Parragh et al. (2010), fragments Rist & Forbes (2021), or zero-sum blocks Malheiros et al. (2021). We design our sequential approach and the respective dispatching algorithm around this property: we create respective zero-sum blocks via the hypergraph matching described in Section 3.1.2 and use these to construct a dispatching graph that allows us to obtain the respective vehicle-to-request dispatching by solving a k-dSPP. The remainder of this section briefly outlines how we create the respective dispatching graph and a polynomial k-dSPP algorithm.

We start by creating our dispatching graph as a weighted and directed source-sink graph $\tilde{\mathcal{G}} = (\tilde{\mathcal{N}}, \tilde{\mathcal{A}}, \theta)$, consisting of a set of vertices $\tilde{\mathcal{N}}$, a set of arcs $\tilde{\mathcal{A}}$, and a vector of weights θ , containing a weight θ_a for each arc $a \in \tilde{\mathcal{A}}$ (see Figure 2). The vertex set $\tilde{\mathcal{N}} = \{w, w^*\} \cup \tilde{\mathcal{K}} \cup \tilde{\mathcal{B}}$, comprises a dummy source w and a dummy sink node w^* , a subset of vehicle vertices $\tilde{\mathcal{K}}$, and a subset of pooled





requests $\widetilde{\mathcal{B}}$ which we refer as blocks. Here, we associate each vehicle vertex with a vehicle's starting position while associating each block vertex with a feasible sequence of pickup and delivery node visits. More formally, a block $b \in \widetilde{\mathcal{B}}$ is a tuple $(\widetilde{\vartheta_b}, \widetilde{e}_b, \widetilde{t}_b)$ comprise of a sequence of visits $\widetilde{\vartheta}$ for a set of requests, such that precedence, capacity and time window constraints are satisfied. Next, \widetilde{l}_b defines the fixed starting time and \widetilde{t}_b the duration to serve $\widetilde{\vartheta}$.

Arcs are constructed as follows: we create an arc between two block vertices if a block-to-block connection does not violate the time window constraint. Similarly, we create an arc between a vehicle and a block vertex if the block can be reached in time by the vehicle. Finally, we connect the dummy source with each vehicle vertex and connect all vehicle and block vertices to the dummy sink.

A path in $\widetilde{\mathcal{G}}$ that starts at dummy source w and ends in dummy sink node w^* represents a feasible vehicle trip. We can directly associate the trip to the vehicle dependent on the vehicle vertex it contains. We conclude the construction by assigning a weight θ_{uv} to each $(u, v) \in \widetilde{\mathcal{A}}$. Herein, we set the arcs that leave the source or enter the sink to zero. Next, we set the weights of vehicle-to-block arcs (v, \widetilde{b}) to $\theta_{vb} = c_{v\widetilde{\vartheta}_b} - \xi \frac{|\widetilde{\vartheta}_b|}{2}$. The second part is the cost traveling from vehicle vertex v to the first visit in b; the second part represents the profit of serving the requests of a block, with ξ being a large profit term, such that serving requests is always preferred. The weights of the remaining block-to-block arcs connecting block b to b' are set to $\theta_{bb'} = c_{\widetilde{\vartheta}_b \widetilde{\vartheta}_{b'}} - \xi \frac{|\widetilde{\vartheta}_{b'}|}{2}$. The first part is the cost of traveling from the last visit in $\widetilde{\vartheta}_b$ to the first visit in $\widetilde{\vartheta}_{b'}$, whereas the second part represents again the profit of serving the requests.

With graph $\widetilde{\mathcal{G}}$ defined, we now briefly discuss the polynomial-time algorithm for the k-dSPP. As proven by Suurballe (1974), it is possible to increase the number of disjoint paths on graph $\widetilde{\mathcal{G}}$ from *i* to *i*+1 by finding the shortest interlacing on a modified graph $\widetilde{\mathcal{G}}'$. Herein, we initialize the algorithm by finding the shortest path using the Bellman-Ford algorithm. Note that θ_{uv} could be negative, depending on ξ . Therefore, we reweight the arcs similar to the well-known Johnson algorithm, using the result from the Bellman-Ford algorithm. Afterward, we continue with a modified Dijkstra algorithm to iteratively find the i+1-disjoint shortest paths until we reach $k = |\widetilde{\mathcal{K}}|$. For a detailed description of the algorithm, we refer to Suurballe (1974) for the original implementation, or the more recent use in Schiffer et al. (2021).

After identifying a set of disjoint shortest paths Π , we generate a complete solution by concatenating the visit sequence in blocks based on the paths and assigning them to the corresponding vehicle represented by vehicle node $u_c \in \widetilde{\mathcal{K}}$. Blocks that are not contained in any path remain unassigned.

3.2. Integrated approach

In this section, we introduce an ILS-based algorithm that allows to take integrated pooling and dispatching decisions. When solving large-scale instances, it is imperative to chose an efficient solution representation and respective evaluations to obtain a performant algorithm. Accordingly, we first focus in Section 3.2.1 on the solution representation and evaluation methods used in our algorithm. We then Focus on the ILS-based algorithm in Section 3.2.2, before detailing a ruin and recreate procedure and further intensification techniques in Section 3.2.3. To keep this paper concise, we focus all discussions on the cost-minimizing objective and detail the respective fleet minimization component in Appendix C.

3.2.1. Solution representation and evaluation

Recall that we represent a solution for our problem as a set of $|\mathcal{K}|$ routes, one for each vehicle. Typically, one stores each route as a list of nodes that denotes the order in which they are visited. Although the number of required lists is known, the length of each list can vary and depends on each instance's properties, e.g., the number of requests or time horizon, and only materializes throughout the search. In the most extreme case, a single route might contain all requests, thus requiring $\mathcal{O}(2|R|)$ memory, such that we obtain an instance-dependent upper bound on a solution's memory requirement by $\mathcal{O}(|\mathcal{K}| \cdot 2|R|)$.

To maintain each list, we exploit that any node is not visited more than once and maintain the following information for each node $i \in \mathcal{N}$: while PRED(i) denotes the predecessor of i, SUCC(i) denotes its successor; ASSIGN(i) yields the vehicle id node i is assigned to (receiving label $|\mathcal{K}| + 1$ if unassigned); EVALFW(i) holds evaluation information up until node i, starting from the vehicle node, whereas EVALBW(i) holds evaluation information starting from node i to the last visit.

The data in EVALFW(*i*) contains the evaluation information of sequence $\{v, \ldots, i\}$ and EVALBW(*i*) holds $\{i, \ldots, \vartheta(-1)\}$, repectively, where $\vartheta(-1)$ is the last visit of route ϑ . We calculate both iteratively using the distance and time window-related formulas of Vidal et al. (2012) and demand calculations analogous to Bulhões et al. (2018) as follows.

Capacity evaluation: Let $q_{sum}(\vartheta)$ be the current number of passengers served, and let $q_{max}(\vartheta)$ be the maximum number of concurrent passengers served in σ . Given two subsequences ϑ^1 and ϑ^2 , we can calculate the concatenated sequence using operation \oplus as follows:

$$q_{\rm SUM}(\vartheta^1 \oplus \vartheta^2) = q_{\rm SUM}(\vartheta^1) + q_{\rm SUM}(\vartheta^2)$$
(3.8)

$$q_{\text{MAX}}(\vartheta^1 \oplus \vartheta^2) = \max\{q_{\text{MAX}}(\vartheta^1), q_{\text{MAX}}(\vartheta^1 + \vartheta^2)\}$$
(3.9)

where a sequence ϑ' containing a single node *i* is initialized as follows:

$$q_{\text{SUM}}(\vartheta') = \begin{cases} 1 & \text{if } i \in P \\ -1 & \text{if } i \in \mathcal{D} \\ 0 & \text{if } i \in \mathcal{K} \end{cases}$$
(3.10)

$$q_{\text{MAX}}(\vartheta') = \max\{0, q_{\text{SUM}}(\vartheta')\}$$
(3.11)

A route ϑ is feasible regarding the capacity constraint if $q_{\text{MAX}}(\vartheta) \leq Q$. Note that the load of a vehicle cannot be negative as long as its route satisfies the respective the precedence constraint (2.2).

Travel time evaluation: Let $T_{\rm T}(\vartheta)$ be the accumulated travel time without potential waiting time, and let $T_{\rm LS}(\vartheta)$, $T_{\rm EC}(\vartheta)$ be the latest start and earliest completion time, respectively; $T_{\rm F}(\vartheta)$ indicates whether the sequence is time windows feasible. Then, we define our concatenation operation \oplus as follows:

$$T_{\mathrm{T}}(\vartheta^1 \oplus \vartheta^2) = T_{\mathrm{T}}(\vartheta^1) + t_{\vartheta^1\vartheta^2} + T_{\mathrm{T}}(\vartheta^2)$$
(3.12)

$$T_{\rm EC}(\vartheta^1 \oplus \vartheta^2) = \max\{T_{\rm EC}(\vartheta^1) + t_{\vartheta^1\vartheta^2} + T_{\rm T}(\vartheta^2), T_{\rm EC}(\vartheta^2)\}$$
(3.13)

$$T_{\rm LS}(\vartheta^1 \oplus \vartheta^2) = \min\{T_{\rm LS}(\vartheta^1), T_{\rm LS}(\vartheta^2) - t_{\vartheta^1\vartheta^2} - T_{\rm T}(\vartheta^1)\}$$
(3.14)

$$T_{\rm F}(\vartheta^1 \oplus \vartheta^2) = T_{\rm F}(\vartheta^1) \wedge T_{\rm F}(\vartheta^2) \wedge (T_{\rm EC}(\vartheta^1) + t_{\vartheta^1\vartheta^2} \le T_{\rm LS}(\vartheta^2)) \tag{3.15}$$

A single-node sequence ϑ' is initialized with $T_{\rm T}(\vartheta') = 0$, $T_{\rm EC}(\vartheta') = e_i$, $T_{\rm LS}(\vartheta') = l_i$, and $T_{\rm F}(\vartheta') = TRUE$. A route ϑ satisfies the time window constraint if for any two subsequences ϑ^1 and ϑ^2 , $T_{\rm EC}(\vartheta^1) + t_{\vartheta^1\vartheta^2} \leq T_{\rm LS}(\vartheta^2)$ holds, i.e., if we can reach the first visit of ϑ^2 after serving ϑ^1 exactly at or before the latest start time of ϑ^2 .

Cost evaluation: Let $C(\vartheta)$ be the accumulated cost of sequence ϑ . Then, we define our concatenation operation \oplus as:

$$C(\vartheta^1 \oplus \vartheta^2) = C(\vartheta^1) + c_{\vartheta^1 \vartheta^2} + C(\vartheta^2)$$
(3.16)

Efficiency: Each concatenation operation detailed above can be executed in $\mathcal{O}(1)$ time. EVALFW(i) and EVALBW(i) can thus be calculated in $\mathcal{O}(|2\mathcal{N}|)$ by traversing the route using SUCC(i), starting from the vehicle node v, to perform EVALFW(SUCC(i)) = EVALFW(i) $\oplus i$, and PRED(i), starting from the last visit $\vartheta(|\sigma|)$, to set EVALBW(PRED(i)) = $i \oplus$ EVALBW(i), respectively. When performing any changes to a route, EVALFW(i) and EVALBW(i) need to be updated accordingly.

For each vehicle $v_j \in \mathcal{K}$, we additionally maintain $\text{LAST}(v_j)$ which holds the information on the last visit of route ϑ^j . The objectives can be obtained as follows: i) $|\{i \in \mathcal{N} : \text{ASSIGN}(i) = |\mathcal{K}| + 1\}|$ is the number of unassigned requests, and ii) $\sum_{i \in \mathcal{K}} C(\text{EVALFW}(\text{LAST}(j)))$ the total travel cost.

3.2.2. Iterative local search

Algorithm 1 shows the pseudocode of our ILS-based metaheuristic. First, we generate an initial solution using a simple but fast parallel insertion construction heuristic (l. 1). Herein, we randomly assign one request to each vehicle and distribute the remaining requests sequentially to their best position in any route.

Algorithm 1: Iterative local search approach

```
Input: Time limit \Omega_A; Average split size \chi; Maximum iterations M_A; Maximum iterations
                    per split solution M_S; Maximum perturbation size Z_A
 1 \sigma \leftarrow \text{parallel_insertion\_construction}();
 2 \sigma', \sigma^* \leftarrow \sigma;
 3 i^{\text{ILS}} \leftarrow 0;
 4 while \Omega_A not reached do
            i^{\text{LS}} \leftarrow 0;
 \mathbf{5}
            T \leftarrow T^{\text{INIT}}:
 6
            while i^{LS} < M_A do
 7
                  P \leftarrow \texttt{decompose}(\sigma', \chi);
  8
                  for \sigma_i in P do
 9
                    \sigma_i \leftarrow \texttt{ruin\_and\_recreate}(\sigma_i, M_S, T, T^{\text{DEC}});
10
                  \mathcal{B} \leftarrow \texttt{extract\_blocks}(P);
11
                  \widetilde{\mathcal{G}} \leftarrow \texttt{create\_graph}(\mathcal{B});
12
                  \Pi \leftarrow \mathtt{run\_kdsp}(\widetilde{\mathcal{G}});
13
                  \sigma'' \leftarrow \texttt{build\_from\_kdsp}(\Pi);
\mathbf{14}
                  \sigma, \sigma', \sigma^* \leftarrow \texttt{accept\_solution\_rnr}(\sigma'', \sigma', \sigma, \sigma^*);
15
                  T \leftarrow T - T^{\text{DEC}} M_S;
16
                  i^{\text{LS}} \leftarrow i^{\text{LS}} + M_S;
17
            \sigma^*, \sigma' \leftarrow \texttt{accept\_solution\_ils}(\sigma'', \sigma', \sigma^*);
18
            \sigma' \leftarrow \texttt{perturb}(\sigma', Z_A);
19
            i^{\text{ILS}} \leftarrow i^{\text{ILS}} + 1;
\mathbf{20}
21 return \sigma
```

Then, we run the main ILS loop until we reach a time limit Ω_A (l. 4). For each ILS iteration, we perform our local search for M_A iterations (l.7–l.17): herein, we first decompose the problem by inspecting the current solution σ' and randomly partitioning its routes such that each part contains χ requests on average. Then, we distribute unassigned requests between the partitions. To do so, we maintain a history comprising two counters tracking i) how often requests are scheduled directly after another and ii) how often requests are assigned to the same route. For each unassigned request, we sum up the counters for each request in a partition and use this sum as weight in a roulette-wheel selection.

Each resulting partition constitutes a subproblem containing the respective vehicle and requests, for which we use the vehicle's sequence in σ' to obtain an initial solution. We then improve these solutions in parallel by using an R&R metaheuristic (l. 10). After a predefined number of M_S iterations, we collect the best solution found for each subproblem and recombine these. Herein, we first identify and extract the block information from the partial solutions (l. 11). For each block, i.e., a sequence of pickup and delivery visits, we obtain the total time spent (service time, waiting time, and travel time), the total distance traveled, as well as the earliest and latest beginning of service time, such that the waiting time is minimal. Note that the block starting time window depends on the partial solutions, resulting in tighter bounds than if computed in isolation.

Next, we create the dispatching graph $\tilde{\mathcal{G}} = (\tilde{\mathcal{N}}, \tilde{\mathcal{A}})$ (l. 12), analog to Section 3.1.3. After solving the k-dSPP in line 13, a set of disjoint shortest paths Π has been produced, which is used in line 14 to generate routes by concatenating the sequences of the corresponding routes until reaching the sink.

During the search, we may allow a decrease in quality when accepting incumbent solutions to balance exploration and intensification (l. 15). Here, we adopted the record-to-record aspiration criterion, which performs well for the classical PDPTW instances (see, e.g., Santini et al. 2018), as follows. New best solutions are always accepted; otherwise, a solution is only accepted if the gap between it and the current best solution is smaller than a threshold T, which decreases linearly by T^{DEC} with each iteration. We use $T^{\text{INIT}} = 0.333$ and $T^{\text{DEC}} = T^{\text{INIT}}/M$ as parameters for the initial temperature and decrement step-value. The R&R procedures run in parallel for each subproblem and start with the current criterion state. We update the temperature after recombining the solutions by decreasing the temperature M_S times (l. 16).

After the local search procedure finishes, we perform a second check for accepting the solution found (l. 18). We accept any new best solution found so far. Otherwise, we either use the current solution found by the cost-minimization component or revert to the best solution. This choice is probability-driven and controlled by two counters, the total number of ILS iterations and the number of ILS iterations i^{ILS} without any improvements to the best solution i^{LAST} , dividing the latter by the former, $i^{\text{LAST}}/i^{\text{ILS}}$.

The perturbation procedure attempts to escape local optima (l. 19) by performing either a random relocation or an exchange operation with a predefined probability α^{ILS} for a total number of Z_A times. This random relocation moves a request to a feasible position in another route, which is again chosen randomly: the exchange operations randomly select two requests in distinct routes and attempt to insert them in the other routes at a random feasible location.

3.2.3. Ruin-and-recreate procedure

The core component of our algorithm follows the R&R approach (Schrimpf et al. 2000). In each iteration, parts of the solution are ruined (l. 4) by removing requests and then recreated (l. 5) by reinserting the unassigned requests again in a cost-improving manner. We follow the implementation of the slack-inducing string removal approach from Christiaens & Vanden Berghe (2020). Herein, the ruin operator is an adjacent string removal operator, which selects an assigned requests randomly and then iteratively removes sequences of visits from routes containing related requests to introduce slack for potentially better re-insertions. The recreate operator is a variant of a simple greedy best

Figure 3: Example of a run of the recombination procedure.



(a) Partial solutions σ^1 and σ^2 .



insertion approach. Unassigned requests are sorted and then sequentially inserted to their best position in any route. A so-called blink mechanic adds diversity to this approach, wherein insertion positions may be skipped with a slight chance. We modified this approach to test the insertion of at most 40 requests per operator call to reduce the computational runtime.

Finally, the R&R procedure stops after a predefined number of M_A iterations has been performed. In the following, we detail the operators and intensification methods applied.

Algorithm 2: Ruin and Recreate
Input: Feasible solution σ ; Maximum iterations M_A ; Initial acceptance threshold T ;
The shold decrement value T^{DEC}
1 $\sigma' \leftarrow \sigma;$
2 $T \leftarrow T^{\text{INIT}};$
3 for $i \leftarrow 1$ to M_A do
4 $\sigma'' \leftarrow \operatorname{ruin}(\sigma');$
5 $\sigma'' \leftarrow \texttt{recreate}(\sigma'');$
6 $\sigma, \sigma' \leftarrow \texttt{accept_solution}(\sigma, \sigma', \sigma'', T);$
7 $T \leftarrow T - T^{\text{DEC}};$
s return σ

Ruin: Adjacent string removal. The ruin operator starts by selecting a request at random, which we refer to as r^* . Let $\overline{R}(r^*)$ be a sorted set of related requests of r^* , including r^* , and let $\operatorname{Assign}(r_i)$ be the current vehicle assignment of the respective pickup node of r_i . For every $r_i \in \overline{R}(r^*)$, the algorithm selects a consecutive sequence of k_s visits from route $\operatorname{Assign}(r_i)$ and either a) and removes the corresponding requests (called split), or b) selects a substring $\overline{\vartheta} \subset \vartheta$, which is ignored, and remove the requests of the remaining consecutive string (called split-string). This process continues with the following requests, skipping routes already modified and terminating if a certain threshold of routes k_s has been considered.

Both the number of strings to remove k_s and the length of the string l_{ϑ} are randomly chosen from a uniform distribution. We bound them using two parameters: i) an average number of nodes to remove \bar{c} , and ii) a maximum number of removed strings L^{max} , using the following equations

$$k_s = \left\lfloor \mathcal{U}(1, \frac{4\bar{c}}{1+l_s^{max}} - 1) \right\rfloor$$
(3.17)

$$l_{\vartheta} = \left\lfloor \mathcal{U}(1, \min\{\left|\vartheta\right|, l_s^{max}\}) \right\rfloor$$
(3.18)

where, $l_s^{max} = \min\{L^{max}, |\overline{\vartheta} \in \mathcal{K}|\}$ holds the maximum string cardinality, which is the minimum of the parameter L^{max} and the average cardinality of the current routes. We use $\overline{c} = 15$ and $L^{max} = 10$ as identified in our parameter tuning (see Appendix B).

We randomly perform the split with probability $\alpha^{\mathbb{R}\&\mathbb{R}} = 0.75$, or split-string sub-procedure otherwise, aiming to remove l_{ϑ} nodes. The split procedure selects a consecutive string of nodes, which are removed. In split-string, we first calculate the length of the substring m, where $l_{\vartheta} + m \leq |\vartheta|$. We start with m = 1 and increment m by one with probability $\beta^{\mathbb{R}\&\mathbb{R}} = 0.10$ until it has either not been incremented or reached a maximum of $|\vartheta| - l_{\vartheta}$. After removal, we check whether requests were only partially removed (either pickup or delivery node) and fully remove them from the solution.

Recreate: Greedy insertion with blinks. To recreate our solution, we use a greedy insertion heuristic. The procedure sequentially tries to insert pickup-and-delivery-pairs, i.e., requests, to their best possible positions, ensuring they satisfy the precedence constraint. Similar to Christiaens & Vanden Berghe (2020), we ignore some insertion positions while searching for the best solution with a low probability. These so-called blinks add randomness to the insertion procedure similar to other noise-based methods from the literature, e.g., Ropke & Pisinger (2006).

The order in which requests are inserted is determined anew every time the procedure is called using a weighted roulette-wheel selection. We use six different criteria (random, far, close, tw-length, tw-start, tw-end) and their corresponding weights (6, 2, 1, 4, 2, 2) to order the unassigned requests based on their related pickup nodes i) at random, ii) descending or iii) ascending by their distance to the closest vehicle starting location, iv) ascending by the time window length, v) ascending by the earliest start time, or vi) descending by the latest start time.

The routes are searched sequentially. First, routes already serving requests are considered. If no insertion position is found – either because it cannot be inserted feasibly or was randomly disregarded by the blinking mechanic – all currently empty routes are considered for insertion, and one with the minimum additional cost is selected.

When traversing a route to look for a feasible insertion point, we consider insertion positions by first considering the pickup node position and then testing all viable subsequent positions for inserting the delivery node. By doing so in a lexicographical order, we can efficiently use the preprocessed evaluation data EVALFW(i) and EVALBW(j), as follows.

When testing an insertion of request r = (i, j) after node i' and before node j' = SUCC(i'), we can use the preprocessed data defined in Section 3.2.1. Figure 4 shows the sequences after insertion if done in a lexicographical manner. The corresponding concatenation operations are shown on the right side using the preprocessed route data. We can observe that the first part of the sequence up

to but excluding the delivery node d is part of the following sequence. By reusing this data, we can test all pickup-and-delivery positions efficiently in $\mathcal{O}(|\vartheta|^2)$.

Figure 4: Sequences considered when inserting request r = (p, d) between nodes i, j and the concatenation operations to evaluate them.

Sequence after insertion	Evaluation using preprocessed data
$\begin{aligned} \vartheta' &= \{\dots, i, p, d, j, j', j'' \dots \} \\ \vartheta'' &= \{\dots, i, p, j, d, j', j'' \dots \} \\ \vartheta''' &= \{\dots, i, p, j, j', d, j'' \dots \} \\ \vdots \end{aligned}$	$\mathrm{EvalFw}(i') \oplus p \oplus d \oplus \mathrm{EvalBw}(j')$ $\mathrm{EvalFw}(i') \oplus p \oplus j \oplus d \oplus \mathrm{EvalBw}(j')$ $\mathrm{EvalFw}(i') \oplus p \oplus j \oplus j' \oplus d \oplus \mathrm{EvalBw}(j'')$

Accepting solutions. During the search, we may allow a decrease in quality when accepting incumbent solutions to balance exploration and intensification (1. 6). Herein, we follow the same linear record-to-record approach described in Section 3.2.2. When encountering a better solution, we always accept it and continue the search.

Intensification procedure. To support intensification, we perform an intra-route local search approach based on Balas & Simonetti (2001) to find efficient itineraries whenever we reach a new best solution. Herein, a restricted neighborhood with visits up to a predefined number of positions, k, from their current location can be exhaustively searched efficiently.

Let k be a positive integer and ϑ be a feasible initial route, where $\vartheta(u)$ denotes the position of node u in route ϑ . The BS(k) neighborhood is defined as all permutations of visits π in ϑ , satisfying

$$\pi(u) < \pi(v) \qquad \qquad \forall u, v \in \mathcal{N} : \vartheta(u) + k \le \vartheta(v) \tag{3.19}$$

where $\pi(u)$ denotes the position of node u in the permutated sequence. Equation (3.19) enforces that permutations respect the maximum number of positions moved by any node, k, from their original position in ϑ . Similar to Balas & Simonetti (2001) the search for the best permutation π satisfying (3.19) reduces to a shortest path problem on an auxiliary acyclic graph $G^* = (V^*, E^*)$. This graph is arranged into layers so that only successive layers are connected by arcs. There are 2|R| + 2 layers required at maximum. The first layer and last correspond to the departure of a vehicle from its starting location, and a dummy node represents the end of the route. The other layers correspond to the 2|R| visits in π . Each layer contains multiple nodes representing distinct states of anticipated and delayed visits to locations in relation to ϑ . More specifically, each node in V^* represents a quadruplet $(i, i + j, S^-(\vartheta, i), S^+(\vartheta, i))$, where i is the current position in π ; j is the offset from the current location, which visit allocated to position i in π , such that $\pi(i) = \vartheta(i + j)$; $S^-(\vartheta, i)$ is the set of visits that are allocated at position i or later in ϑ and allocated at position ior before in π ; and, finally, $S^+(\vartheta, i)$ is the set of visits allocated before position i in ϑ that will be allocated at position i or after in π .

Figure 5: Graph G^* with k = 3 and $\vartheta = \{v, p_1, p_2, d_1, p_3, d_2, d_3\}$ (Nodes contain the corresponding pickup or delivery node for visualization). Arcs in black show the path of permutation $\pi = \{v, p_1, d_1, p_3, p_2, d_3, d_2\}$. Note that the dashed arc from Layer 1, Node 1 to Layer 2, Node 4 is invalid as the resulting sequence would violate the precedence constraint.



Observe that $S^{-}(\vartheta, i)$ and $S^{+}(\vartheta, i)$ provide enough information to restrict the search to solutions satisfying Constraints (3.19), by restricting the search to nodes such that all visits in $S^{+}(\vartheta, i)$ are located in no more than k positions earlier than v in ϑ . The precedence constraint can be trivially checked by maintaining the information of allocated pickup nodes in each node and prohibiting traversing invalid arcs.

Figure 5 shows G^* on a simple example with three requests, starting with route $\vartheta = \{v, p_1, p_2, d_1, p_3, d_2, d_3\}$ and k = 3. Without considering capacity or time window constraints, finding the shortest path in this graph can be done in $\mathcal{O}(k^2 2^{k-2}|\vartheta|)$ time. Note that graph G^* need only to be built once – requiring $\mathcal{O}(k2^{k-2}n)$ space – and can be reused. Furthermore, by fixing k, we obtain a polynomial-time algorithm to locate the best permutation.

Time window constraints cannot be included in the graph structure and need to be checked during the search, rendering the problem of finding the shortest path with resource constraints. This problem is NP-hard (Irnich & Desaulniers 2005) and requires a labeling algorithm that no longer runs in polynomial time. Each vertex in the graph now may contain multiple labels, each representing a non-dominated path through the graph G^* . We reuse the evaluation mechanics from Section 3.2.1 to define a label $L(v^*, \vartheta') = (C(\vartheta'), q_{\text{SUM}}(\vartheta'), q_{\text{MAX}}(\vartheta'), T_{\text{T}}(\vartheta'), T_{\text{EC}}(\vartheta'), T_{\text{LS}}(\vartheta'))$, where v^* is the corresponding vertex in G^* and ϑ the sequence of visits reaching and including the node added at v^* . Note that each label at any vertex v^* visited the same nodes in \mathcal{N} but in a different order. When extending a label L to a compatible successor, we first check whether the new sequence is feasible regarding the capacity and time window constraint. If so, we create a new label for the extended sequence, add it to the corresponding label set, and perform a dominance check. Given two labels $L^1(v^*, \vartheta^1)$ and $L^2(v^*, \vartheta^2)$, we say L^1 dominates L^2 if all the following hold:

$$C(\vartheta^1) \le C(\vartheta^2) \tag{3.20}$$

$$T_{\rm EC}(\vartheta^1) \le T_{\rm EC}(\vartheta^2) \tag{3.21}$$

While Equation (3.20) ensures that we keep the lowest cost label, 3.21 ensures that a label where we arrive earlier stays as well, as further extensions may invalidate labels with later arrivals due to violations in the time window constraint. As the number of labels per vertex can grow exponentially due to the potential trade-off in cost and time, we limit the number to only consider the four leastcost labels. Note that this adaption may lead to searches terminating without finding any feasible solution.

4. Experimental Design

In this section, we first introduce our case-study data and instances. We then generate mid-sized benchmark instances for our algorithmic component analysis.

We implemented the algorithms presented in Section 3 in the Rust programming language, compiled using rustc version 1.70 in release mode and with link-time optimization. The experiments were conducted on a cluster setup consisting of Intel i9-9900 processors with 16 cores running at 3.10 GHz and 64GB RAM, using Ubuntu 20.04 LTS operating system. We limit our runs to four cores and 8GB of RAM to allow conducting our experiments in parallel. Furthermore, we run experiments with five different seeds to reduce the probability of outliers.

The source code, results, and documentation on building and running the algorithm are available online at https://github.com/tumBAIS/LargeScalePDPTW.

4.1. Case-study instances

One of the largest and most regularly used data sources in the literature is the data from the NYC Taxi and Limousine Commission (New York City Taxi and Limousine Commission 2015), which provides historical taxi trip data. We use a preprocessed version of these trip data by Jungel et al. (2023), available online at https://github.com/tumBAIS/ML-CO-pipeline-AMoD-control. This data includes origin and destination location information as well as timestamps. We use OpenStreetMap (OSM) network data to map the trip data and calculate travel time and distances, assuming an average travel speed of 20 km/h. Pickup and dropoff locations are matched to the nearest node of the street network (see Figure 6a). The distance and travel time matrix is preprocessed once for the whole network using an all-pair shortest path algorithm.

Figure 6b shows the 10-minute rolling average of trips arriving throughout the day in our sample week, spanning January 5th to January 11th in 2015. We observe similar patterns for weekdays

Figure 6: New York City trip and network data.



(a) New York City street network from OSM with pickups and dropoff points marked in red and blue, respectively.

(b) Arriving trips per minute during the sample week (10minute rolling average). Selected time frames are highlighted in green.

(Mon-Fri) and diverging patterns for the weekend days (Sat, Son), e.g., at 2 a.m. For this reason, we generate our large-scale instances only for weekdays and extract trip data for six distinct time frames for one hour each: night (2-3 a.m.), early morning (6-7 a.m.), late morning (10-11 a.m.), afternoon (2-3 p.m.), late afternoon (6-7 p.m.), late evening (10-11 p.m.).

Requests are generated from the trip data of a given day and time frame. We use the dropoff time of the trip data as the fixed point and calculate the corresponding (earliest) pickup time based on the direct travel time, i.e., $e_p = e_d - t_{pd}$. The latest begin-of-service times l_p, l_d are implicitly defined by the buffer δ .

Vehicles start at random trip destinations, sampled from the full data set. Capacities of vehicles are set to three requests when pooling and one for taxi operations. We consider a base scenario with a fleet of 1000 vehicles. Table 1 shows the number of requests in the generated instances.

Date	Day	2-3h	6-7h	10-11h	14-15h	18-19h	22-23h
2015-01-05	Mon.	1.385	6.742	11.457	13.094	18.736	10.070
2015-01-06	Tue.	1.035	7.085	11.922	13.441	19.490	12.150
2015-01-05	Wed.	1.669	7.296	14.086	15.256	21.375	15.633
2015-01-08	Thu.	1.815	7.705	15.126	15.212	21.170	17.115
2015-01-09	Fri.	2.869	6.878	13.552	14.820	21.222	18.691

Table 1: Number of requests for the instances generated from the NYC data set.

Our experiments are designed to capture the marginal benefits of transitioning from classic taxi operations to flexible ride-pooling offerings; see Figure 7. In the first setting (A), we do not allow any deviation from the desired pickup and dropoff time, emulating the current operations of taxi fleets. Nevertheless, we consider ride-pooling in this setting to identify (if any) on-route pooling opportunities. In the second setting (B), we consider deviating from the desired dropoff time by

Figure 7: Time window settings considered in the case study (A: fixed pickup and dropoff; B: fixed pickup, variable dropoff; C: variable pickup and dropoff).



 δ minutes, $\delta = \{1, 2, 3, 4, 6, 8, 10, 12, 15, 20\}$. This does not impact taxi operations as the pickup time is still fixed. However, ride-pooling operations may benefit from this increased flexibility, allowing more requests to be served concurrently. In setting (C), which is the original problem setting (see Section 2), we extend the time windows for pickup and dropoff, allowing for even more flexibility on the operator side. For this setting, we conduct our experiments with taxi operations and ride-pooling, as the former may also benefit from the flexibility.

Note that solutions found for a buffer setting δ_a are feasible for δ_b where $\delta_a < \delta_b$. We exploit this property in our managerial experiments as follows. We start by solving the most restrictive case of zero buffer ($\delta = 0$) with a runtime of 8 hours for each day and time frame. All subsequent settings with $\delta \ge 0$ s are run for 4 hours but warm-started using the solution of the previous buffer setting. With this approach, we avoid issues arising from the heuristic nature of the algorithm. Note further, that for taxi operations with no flexibility, we only perform a single run per instance, as the k-dSPP always finds the optimal solution in this setting.

4.2. Mid-sized benchmark set

We create two additional sets comprising mid-sized instances following the instance generation protocol detailed in Section 4.1 – one for our computational study comparing the sequential and integrated approach and one for algorithmic tuning and analysis. In both cases, we consider a subset of requests for the same time horizon of one hour. To ensure the temporal properties of the large-scale instances are retained, we do not select a subset of requests at random but equally distanced.

Our first set, MID²⁵, comprises the period 18-19h only, where we select 25% of the original requests. These instances comprise around 5000 requests. Here, the aim is to compare the sequential, integrated, and combined approaches on high-demand instances. We consider buffer values from $\delta = 1, \ldots, 6$ minutes to analyze their performances on varying degrees of freedom. Furthermore, we compare three fleet size configurations: a) a low availability setting with 200 vehicles, b) availability of 500 vehicles, where we have roughly one vehicle per 10 requests, and c) high availability of 800 vehicles.

The second set is created for analysis and tuning of the metaheuristic approach. For set MID^{10} , we select 10% of the original requests and omit the early morning periods of 2–3 a.m. and 6–7 a.m. due to their small size. We consider all buffer settings as in the large instances and generate three different fleet size configurations with 100, 200, and 300 vehicles, respectively. From this setting, we randomly select one instance per day, resulting in 18 instances in total.

5. Results

In the following, we discuss our numerical results. First, we focus on a pure algorithmic perspective and compare the performance of our proposed algorithms: the sequential matheuristic (MATH), the iterated local search that takes integrated decisions (ILS), and our ILS when using the solution of MATH as a warmstart (COMB). Second, we compare our developed algorithm against a known benchmark data set from the literature that provides instances for the PDPTW in a medium to large-scale ride-hailing context. Finally, we use our best-performing algorithm to derive managerial insights for the presented large-scale case study.

5.1. Computational Analyses

Figure 8 shows the performance of MATH, ILS, and COMB over the set of mid-sized instances MID²⁵ with 25% of the total requests for different fleet sizes. As can be seen, ILS and COMB outperform MATH on our primary objective, which is to minimize the number of unassigned requests, see Figure 8a. Figure 8b shows our secondary objective which is to minimize the total traveled distance. Here, it is not possible to provide a direct comparison as the total traveled distance could only be compared for the same primary objective values. However, it is worth notifying that COMB, which performs equal or better than ILS on our primary objective, also outperforms ILS on the secondary objective.

Beyond these objective-focused observations, we identify two additional effects: in settings with an abundant supply of vehicles, all algorithms yield a similar performance on the primary objective. As soon as vehicle availability becomes a bottleneck, we observe a significant performance difference between the studied algorithms. Analyzing Figure 8a, we further observe that the share of unassigned requests remains too high from a practical perspective as an operator would aim at expanding its fleet size to provide a better service level. With these observations in mind, we focus the remaining discussions on the setting with a fleet size of 500 vehicles as i) the setting with 800 vehicles does not allow for meaningful algorithmic studies as it compensates bad algorithmic decisions by abundant vehicle supply, and ii) the setting with 200 vehicles leads to unrealistically high rates of unserved requests from a practitioners perspective.

Figure 9 and 10 show the search progress over the runtime for each buffer setting $\delta = 1, \ldots, 6$. The result of MATH, which finishes in a few minutes, is shown as a horizontal line for comparison. The combined approach, COMB, is warm-started with the solution from MATH. In terms of unassigned requests, we observe that the integrated metaheuristic ILS reaches and improves the solution of



Figure 8: Computational study final results per fleet size.

MATH in at most 10 minutes on average. COMB is able to improve on the sequential solution and

produces better results than ILS approach in the runtime of one hour. However, the difference is getting small with higher buffer settings $\delta \geq 3$. In any case, COMB converges faster to its final value then ILS.

Focusing on the total distance traveled, we observe an increase during the search in COMB when started with the solution from MATH. This effect correlates with a decrease in unassigned requests; however, we see stagnation after a few minutes, i.e., even though improvements in terms of unassigned requests are found, the total distance traveled by the fleet does not increase significantly. This implies that the metaheuristic is able to find more efficient routing decisions, which is especially visible for higher buffer settings, e.g., $\delta \geq 5$, where the total distance almost reaches initial solution found by MATH. Remarkably, the COMB approach succeeds in serving more customers at a lower totaled traveled distance compared to the ILS approach.

Figure 11 shows the final solutions for the instances with 500 vehicles, separated by the buffer length δ . We see a similar performance of ILS and COMB for unassigned requests. MATH, however, seems to stagnate with higher buffer settings. In the case of total distance traveled by the fleet, we can see that MATH produces solutions with less distance traveled. This is not surprising, as these solutions serve less requests than the solutions found by the metaheuristic. Furthermore, we can observe that the best performing COMB approach does tend to produce solutions with not only more requests served but less total distance traveled by the fleet.

In summary, we see MATH produces good solutions for scenarios with large fleets, where most of the requests can be served. In these cases, improving these solutions using our metaheuristic (COMB) may not result in significant improvements, especially with higher buffer settings (for



Figure 9: Search progress (best solution) over time regarding unassigned requests per buffer setting for 500 vehicles.

Figure 10: Search progress (best solution) over time regarding total distance traveled per buffer setting for 500 vehicles.



more details, see Appendix A). For the under-saturated case, where the fleet is sparse, we can see that solutions from MATH are outperformed by ILS. Comparing the results of the metaheuristic ILS with the default initial solution generation and warm-started with the solution by MATH, i.e., COMB, we can see that the approach benefit from a good initial solution, resulting in better final solutions -6.8% fewer unassigned requests on average - after a time limit of just one hour.



Figure 11: Computational study final results per buffer length.

6. PDPTW benchmark

We evaluate the efficacy of our approach by comparing its performance solving PDPTW instances. Sartori & Buriol (2020) recently created a new benchmark based on real-world street network data of Barcelona, Berlin, New York City, and Porto Alegre, ranging from 100 to 5000 nodes (50 to 2500 requests). Each consists of 25 instances, 5-7 per city. Current best-known solutions are curated by the benchmark creator and available online at https://github.com/cssartori/pdptw-instances.

The classic PDPTW uses a hierarchical objective, where we prioritize minimizing the number of vehicles used before the total distance traveled. Furthermore, no unassigned requests are allowed in a feasible solution. Herein, we include a fleet minimization component to our ILS-based metaheuristic approach. We refer to Appendix C for details.

Note that we limit our discussions to the benchmark of Sartori & Buriol (2020). In a preliminary analysis, we looked at the structure of the best solutions produced in the literature for the Li & Lim (2001) and Sartori & Buriol (2020) benchmarks. We assessed their similarity to our large-scale ridesharing case (see Appendix D). Our findings show that the classical Li & Lim (2001) instances tend to have a higher tendency to result in routing decisions with long and sometimes route-spanning blocks. In contrast, solutions for the Sartori & Buriol (2020) benchmark exhibit a higher rate of blocks assigned to routes. This property overlaps with the solutions resulting from our large-scale ride-sharing instances. To this end, we focused on the Sartori & Buriol (2020) benchmark instances for our comparative analysis.

Table 2 compares our results with the literature for the Sartori & Buriol (2020) benchmark. As is usually done in the PDPTW literature, we report our results as accumulated values (the number of vehicles and total cost, i.e., distance traveled) per instance size. Note that the bestknown solutions reported on https://github.com/cssartori/pdptw-instances include unpublished work without any additional information regarding runtime and hardware, and a recent contribution by Vadseth et al. (2023), which improved 203 instances by warm-starting their approach using the previously best-known solution. To our knowledge, the only published results produced without using best-known solutions are from Sartori & Buriol (2020).

Our results show the competitiveness of our approach, especially for larger instance sizes, where we improved on the previous results by 3.1% in terms of the cumulative number of vehicles. Furthermore, we contribute to the research by providing 107 new best solutions, with 90 reducing the number of vehicles.

	I	BKS		SB^*		HS*	I	IS**
Instance	Veh	Cost	Veh	Cost	Veh	Cost	Veh	Cost
n100	164	25.264	164	25.388	164	25.349	164	25.326
n200	332	46.617	337	46.587	332	46.846	332	46.686
n400	585	85.277	589	85.887	589	87.604	587	86.834
n600	828	122.156	840	122.130	838	126.081	834	124.996
n800	1134	163.806	1150	163.341	1147	170.952	1147	169.756
n1000	1383	222.846	1401	226.228	1389	232.096	1385	230.305
n1500	2078	297.367	2115	303.478	2082	313.057	2076	311.215
n2000	2861	412.036	2924	425.343	2902	438.839	2886	433.970
n2500	3135	493.731	3201	506.436	3149	519.504	3128	513.210
n3000	4137	583.567	4276	625.072	<u>4130</u>	614.621	4105	605.405
n4000	5534	761.630	5944	866.430	5605	820.816	5541	796.389
n5000	6262	918.989	6802	1.089.677	6443	1.029.592	6335	982.320

Table 2: Sartori & Buriol (2020) benchmark results (cumulative values for the number of vehicles and cost). We mark results in bold if they improve the results reported by Sartori & Buriol (2020), and underline results improving on the previously best-known solutions.

BKS: best-known solutions from https://github.com/cssartori/pdptw-instances

(last-access: 2024-04-26); SB: Sartori & Buriol (2020); HS: this work

* time limit of 5, 15, 15, 30, and 60 minutes for n100, n200, n400, n600, and n800-n5000

** extended time limit of 15, 45, 45, 90, and 180 minutes for n100, n200, n400, n600, and n800-n5000

6.1. Managerial Analyses

In the following, we use our ILS to study the large-scale instances introduced in Section 4.1 from a managerial perspective. In this context, we discuss the impact of fleet-sizing and delays incurred on the customer side.

Comparison of fleet variations to ride-sharing flexibility. We evaluate the benefits of introducing flexibility for the service level by comparing our results to a commonly used – although expensive – alternative: increasing the fleet size. We focus on a single day and consider fleets of size 1000 to 2000

vehicles, with 200-vehicle increments. Figure 12 shows the corresponding results for four scenarios: a) the aforementioned fleet increase of the classic taxi service, b) taxi operations with possible delayed pickup (and delivery), c) ride-sharing with a fixed pickup time, but variable delivery, and d) ride-sharing with flexible pickup and delivery time. We observe a slower growth in service level when the fleet size increases, whereas increased allowed delay introduces more considerable gains with lower values and diminishing increases with larger values. For the most flexible option (ride-





sharing with variable pickup and delivery time up to δ minutes), we see that even a small buffer of two minutes allows a similar service level than an increase of the fleet by 600 additional vehicles for the time frame starting at 6 p.m. (18h).

Buffer effects on ride-sharing operations. We now continue with the most flexible case, i.e., ridesharing with increasingly flexible pickup and delivery times. Figure 13 shows the service level – the percentage of requests served by the fleet – for increasing buffer levels δ , i.e., increasing flexibility of arrival delays. Given the fleet limitation of 1000 vehicles, not all requests can be served for dense time-frames, e.g., 6 p.m. (18h). We can see a larger relative gain in service level with smaller allowed delays (buffer). Even a two-minute delay allows for an increase from approximately 60 % to 80%, as shown for 10 a.m. (10h) and 2 p.m. (14h). We further observe a larger spread in the service level for the latest time frame at 10 p.m. (22h). This effect originates primarily from the results of the Thursday and Friday instances, which tend to compose of more a higher frequency of requests compared to Monday to Wednesday.



Figure 13: Service level changes with increasing allowed delay (buffer) for weekdays per hour of the day with a fixed fleet of 1000 vehicles.

Figure 14 shows the distance traveled per request served to increase the flexibility of arrival delays. We can observe, as expected, that in the lower density time-frames at 10 a.m. (10h) and 2 p.m. (14h), the distance traveled per request served is larger than for the dense evening time frame at 6 p.m. Furthermore, we see an increase in average distance traveled when introducing flexibility; however, this effect does not necessarily continue but stagnates after the 2-minute allowed delay (see 18h) or even decreases for the early time frame (10h). Comparing the distance progression with the service level changes in Figure 13, we can see no clear correlation between those indicators, especially for the dense time frame, with roughly similar average distance values of 800 meters for 2 to 20-minute flexibility, with an increase of 10 % of service level for the same range.

Figure 15 shows the effects of introducing ride-sharing and the induced allowed delay on the experienced delay of customers. The average experienced delay is always below half the permitted delay and settles at around 6 minutes for the highest delay setting of 20 minutes. For the high-density time frames at 6 p.m., we see a lower increase in experience delay with higher permitted delay settings.



Figure 14: Distance traveled per request served with increasing allowed delay (buffer) for weekdays per hour of the day with a fixed fleet of 1000 vehicles.

Figure 15: Average delay per customer due to introduced flexibility (buffer) for weekdays and grouped by hour of day with a fixed fleet of 1000 vehicles.



7. Conclusion

In this paper, we presented an algorithmic framework to solve very large-scale instances of an PDPTW variant arising in the context of urban ride-sharing systems. Herein, we developed three approaches: i) a decomposition-based matheuristic, which can solve instances with up to 5000

requests in a few minutes; ii) a ILS-based metaheuristic to handle very large-scale instances with more than 20 thousand requests; and iii) a hybrid approach, where we warm-start the metaheuristic with the solution from the matheuristic.

We conducted a thorough computational study using this algorithmic framework to derive insights into the characteristics of the proposed algorithms. We saw that the metaheuristic improved on the matheuristic within a time limit of 15 minutes. The warm-started hybrid approach produced better results, with up to 6.8% fewer unassigned requests on average. Next, we compared our metaheuristic approach to results for the PDPTW benchmark data set comprising instances with up to 2500 requests. The results showed the algorithm's competitiveness and found 107 new best solutions, with 90 solutions reducing the number of vehicles. Finally, we applied our algorithm to very largescale instances with up to 21375 requests. Our results show that by requesting passengers to allow flexibility in their delivery by just 2 minutes, the mobility provider can operate on a comparative service level as with a 50% larger fleet. Additionally, on average, the passengers would only delay their arrival by less than a minute.

Future research may expand on the instances' scale by using only a subset of trips but expanding the time frame to multiple hours or a full day. Herein, new challenges for the decomposition approach may arise regarding split and recombination policy.

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Appendix A Computational study for the hypergraph matching

In Section 3.1, we described the best-performing hypergraph matching approach we found in our research. In the following, we provide more details on the variations we tested, the experiments conducted, and the results obtained. We start by introducing a simple greedy procedure for selecting a matching. Next, we provide a weighted set partitioning (WSP) formulation based on the WSP formulation from Section 1a. Next, we show the alternative fractional selection policy, based on randomized rounding. Finally, we conclude this section by providing detailed results on the performance of these approaches, which justifies the final selection for our matheuristic approach.

A.1 Greedy matching

We initially considered a naive approach to handle the matching process. Herein, we first simply sort all hyperedges by one of the weight functions. Then, we traverse this list and pick the next best item if none of the associated requests have been part of a previously selected hyperedge. Finally, we stop if all requests are part of selected hyperedges, or we reach the end of the list.

A.2 Weighted set partitioning problem

A matching \mathcal{M} for hypergraph \mathcal{H} is defined as a subset of hyperedges $\mathcal{M} \subset \mathcal{E}$ where requests are only part of at most one hyperedge. To find \mathcal{M} , we tested different approaches. Apart from the greedy selection approach, we considered two two-step approaches. One is the formulation as a WSC problem, as described in Section 3.1.2. In the other approach, we solve a continuous WSP problem to select a subset of hyperedges that may be connected to the same request nodes, i.e., a fractional matching $\overline{\mathcal{M}}$. In a second step, we derive matching \mathcal{M} based on the solution of the WSP using either a greedy selection procedure, or a randomized rounding approach.

We can formulate the WSP by replacing (3.6) from Section 3.1.2 with

$$\sum_{\varepsilon \in \mathcal{E}'} a_{r\varepsilon} x_{\varepsilon} = 1 \qquad \qquad \forall r \in R \qquad (A.1)$$

Note that in preliminary experiments we tested an ILP formulation, to avoid fractional solutions. However, run times of over one hour turned out to be impractical, and the solutions obtained did not considerable improve on the two-step approach.

Selection from fractional solution. When solving the relaxed WSC (WSP) problem, we need to deal with fractional solution values. We test two strategies: a) a simple greedy selection procedure (see Section 3.1 and b) a standard randomized rounding scheme.

The randomized rounding scheme follows the standard technique introduced by Raghavan & Thomson (1987). Herein, a cover (i.e., a pooling option) is selected at random with a probability defined by its fractional solution value. The resulting selection may cover requests multiple times.

We enforce non-overlapping pooling options by using a simple repair procedure, where the request will be removed from all but one of the selected options.

Components	GREEDY	$\mathrm{WSC}_{\mathrm{greedy}}^{\mathrm{LP}}$	$\mathrm{WSC}_{\mathrm{RR}}^{\mathrm{LP}}$	$\mathrm{WSP}_{\mathrm{greedy}}^{\mathrm{LP}}$	$\mathrm{WSP}_\mathrm{RR}^\mathrm{LP}$
Greedy matching	х				
Set Covering		x	х		
Set Partitioning				х	x
Greedy selection		х		х	
Randomized rounding			х		х

Table 3: Summary of the proposed pooling selection procedures.

A.3 Results

We test our approach on mid-sized instances, considering 25% of trips, for the high-volume time frame of 18-19h. Our instances consist of approximately 5000 requests. Note that we preprocess the request pooling options. This process requires several hours of computation, espacially for larger settings of δ . For this reason, we limited $\delta \leq 6$. Furthermore, we do not report any results on a setting of $\delta = 0$, as the prepocessing showed that no pooling option with two or more requests could be found.

In Table 4 we show the average results over all instances. These results indicated that the naive greedy selection procedure outperforms the WSC and WSP approaches. Finally, Table 5–Table 11 show the results for each fleet size setting separately.

Table 4: Comparison of the matching approaches averaged over all fleet sizes (200,...,800) buffer settings $\delta = 1, ..., 6$ (U/Cost).

ω	ρ	GREEDY	$\mathrm{WSC}_{\mathrm{greedy}}^{\mathrm{LP}}$	$\mathrm{WSC}_\mathrm{RR}^\mathrm{LP}$	$\mathrm{WSP}_{\mathrm{greedy}}^{\mathrm{LP}}$	WSP_{RR}^{LP}
ω^1		1180.0/7194.355	1334.6/7385.635	1438.9/7467.493	1311.7/7341.306	1373.3/7466.447
	0.0	870.7/6838.231	890.7/6703.879	1018.5/7017.773	895.4/6708.495	1021.9/7040.573
	0.1	871.4/6838.329	888.4/6701.300	1016.6/7012.572	894.0/6701.676	1014.9/7043.129
	0.2	871.6/6836.022	887.2/6699.581	1014.8/7012.429	891.9/6699.279	1013.0/7031.272
	0.3	873.1/6842.598	885.6/6701.048	1010.2/7009.539	890.6/6701.106	1008.7/7036.593
	0.4	875.7/6845.000	882.9/6698.400	1003.1/7004.935	888.0/6699.360	1005.3/7041.258
$\omega^4(ho)$	0.5	883.1/6854.974	881.5/6701.853	1001.5/7009.856	886.8/6697.925	1001.9/7026.194
	0.6	891.4/6878.164	879.6/6710.580	999.1/6996.288	883.8/6709.140	994.2/7028.833
	0.7	908.0/6916.147	880.1/6713.553	993.5/6995.961	884.2/6715.881	995.3/7024.404
	0.8	942.4/6968.142	881.7/6737.104	988.1/6990.832	886.1/6733.913	986.0/7020.154
	0.9	1017.0/7097.965	891.5/6778.400	974.4/6984.691	890.2/6777.310	973.0/6992.534
	1.0	1153.5/7254.055	1180.3/7196.074	1675.3/7336.007	1180.0/7194.355	1675.5/7335.932

Appendix B Parameter tuning

In this section, we detail our parameter tuning process for the metaheuristic approach presented in Section 3.2. We conducted incremental experiments in a one-factor-at-a-time (OFAT) fashion

ω	ρ	GREEDY	$\mathrm{WSC}_{\mathrm{greedy}}^{\mathrm{LP}}$	$\mathrm{WSC}_\mathrm{RR}^\mathrm{LP}$	$\mathrm{WSP}_{\mathrm{greedy}}^{\mathrm{LP}}$	WSP_{RR}^{LP}
ω^1		2985.5/3278.944	3069.4/3322.791	3090.8/3361.338	3039.9/3322.971	3041.1/3351.332
	0.0	2489.9/3195.219	2620.8/3254.686	2685.7/3300.255	2648.5/3260.037	2717.7/3299.972
	0.1	2490.0/3195.425	2618.0/3254.441	2684.3/3300.273	2647.2/3253.786	2714.2/3294.868
	0.2	2490.4/3196.507	2618.0/3250.695	2683.0/3295.981	2644.9/3254.973	2710.3/3297.761
	0.3	2492.7/3198.575	2615.7/3255.910	2682.9/3296.203	2642.3/3257.690	2708.6/3304.740
	0.4	2497.5/3198.114	2610.7/3262.320	2675.2/3309.270	2640.8/3261.455	2703.8/3311.012
$\omega^4(\rho)$	0.5	2505.7/3208.025	2608.4/3265.880	2676.0/3305.797	2638.6/3266.643	2702.4/3304.229
	0.6	2522.3/3209.043	2608.3/3273.450	2674.5/3310.582	2636.7/3271.027	2696.7/3310.412
	0.7	2555.1/3219.217	2611.4/3276.985	2676.4/3310.786	2639.7/3271.269	2703.4/3310.447
	0.8	2605.6/3225.229	2616.3/3289.606	2676.7/3314.049	2639.7/3290.767	2698.2/3318.870
	0.9	2714.2/3234.905	2630.3/3298.426	2675.4/3312.252	2645.4/3298.910	2691.4/3301.927
	1.0	2914.1/3240.492	2970.8/3281.692	3214.6/3111.353	2965.2/3284.368	3213.6/3111.950

Table 5: Comparison of the matching approaches for the 200 vehicle instances, averaged over buffer settings $\delta = 1, ..., 6$.

Table 6: Comparison of the matching approaches for the 300 vehicle instances, averaged over buffer settings $\delta = 1, ..., 6$.

ω	ho	GREEDY	$\mathrm{WSC}^{\mathrm{LP}}_{\mathrm{greedy}}$	$\mathrm{WSC}_{\mathrm{RR}}^{\mathrm{LP}}$	$\mathrm{WSP}_{\mathrm{greedy}}^{\mathrm{LP}}$	WSP_{RR}^{LP}
ω^1		2186.1/4818.741	2323.8/4870.416	2368.3/4922.043	2293.9/4867.301	2309.1/4917.445
	0.0	1640.0/4665.494	1749.3/4759.670	1856.7/4833.735	1777.8/4761.643	1887.0/4831.570
	0.1	1640.7 / 4669.768	1745.1/4760.180	1855.1/4828.085	1775.4/4758.757	1879.9/4837.343
	0.2	1642.3/4662.946	1744.3/4759.608	1849.6/4827.459	1772.2/4762.537	1875.3/4830.555
	0.3	1643.1/4671.965	1744.5/4756.601	1847.8/4837.390	1771.1/4758.564	1875.2/4839.483
	0.4	1649.1/4672.658	1738.6/4763.288	1840.5/4838.725	1768.1/4764.814	1870.4/4840.993
$\omega^4(\rho)$	0.5	1660.9/4682.427	1736.5/4767.726	1839.8/4842.356	1767.2/4767.261	1863.7/4843.515
	0.6	1678.0/4690.973	1735.8/4778.772	1837.8/4848.617	1763.5/4776.552	1858.2/4854.202
	0.7	1708.5/4717.316	1737.7/4786.482	1834.3/4856.879	1765.9/4783.888	1866.1/4839.318
	0.8	1765.5/4727.801	1741.4/4802.599	1833.3/4852.144	1767.2/4797.778	1855.3/4859.506
	0.9	1889.9/4757.937	1757.0/4821.188	1825.3/4852.615	1769.7/4823.612	1843.0/4842.762
	1.0	2127.1/4765.809	2173.0/4820.209	2549.8/4596.665	2167.0/4825.327	2548.5/4594.403

Table 7: Comparison of the matching approaches for the 400 vehicle instances, averaged over buffer settings $\delta = 1, ..., 6$.

ω	ho	GREEDY	$\mathrm{WSC}^{\mathrm{LP}}_{\mathrm{greedy}}$	$\mathrm{WSC}_\mathrm{RR}^\mathrm{LP}$	$\mathrm{WSP}_{\mathrm{greedy}}^{\mathrm{LP}}$	WSP_{RR}^{LP}
ω^1		1532.6/6201.295	1701.0/6266.906	1745.8/6400.404	1654.5/6267.748	1669.1/6392.717
	0.0	981.8/6068.406	1054.0/6138.428	1196.0/6270.627	1080.8/6143.080	1221.1/6276.299
	0.1	980.9/6072.116	1050.9/6138.227	1192.8/6273.384	1077.8/6147.392	1216.0/6278.538
	0.2	981.1/6073.267	1049.5/6134.004	1187.2/6277.110	1075.8/6145.362	1213.1/6268.704
	0.3	984.0/6074.576	1046.2/6143.305	1185.3/6274.331	1073.8/6146.142	1208.2/6286.012
	0.4	988.2/6080.514	1043.7/6142.247	1177.8/6274.005	1068.0/6157.526	1204.4/6284.090
$\omega^4(ho)$	0.5	1002.2/6083.441	1040.2/6152.210	1176.8/6271.122	1067.7/6150.915	1199.1/6281.223
	0.6	1016.8/6099.991	1037.2/6166.595	1172.0/6281.990	1061.4/6171.651	1189.7/6290.304
	0.7	1047.0/6119.708	1040.5/6163.533	1169.5/6277.818	1062.5/6170.538	1192.6/6286.092
	0.8	1105.9/6130.786	1044.2/6184.225	1162.2/6290.867	1067.0/6184.524	1180.4/6306.789
	0.9	1234.7/6162.225	1061.9/6215.387	1152.1/6293.486	1072.4/6226.208	1165.1/6296.625
	1.0	1489.3/6165.207	1521.1/6206.850	1983.5/6069.544	1515.6/6212.933	1982.3/6064.973

ω	ρ	GREEDY	$\mathrm{WSC}_{\mathrm{greedy}}^{\mathrm{LP}}$	$\mathrm{WSC}_\mathrm{RR}^\mathrm{LP}$	$\mathrm{WSP}_{\mathrm{greedy}}^{\mathrm{LP}}$	$\mathrm{WSP}_\mathrm{RR}^\mathrm{LP}$
ω^1		946.0/7565.772	1118.2/7719.513	1217.5/7818.608	1077.9/7710.161	1144.0/7809.088
	0.0	493.1/7347.679	504.0/7432.558	691.2/7571.052	523.8/7455.587	713.6/7563.534
	0.1	492.9/7353.706	500.1/7433.571	688.4/7562.421	522.3/7449.192	703.9/7590.416
	0.2	493.2/7351.611	496.6/7441.297	686.2/7562.654	519.1/7446.008	705.0/7571.226
	0.3	496.2/7357.245	494.5/7443.295	679.6/7569.114	518.4/7452.287	696.0/7584.730
	0.4	499.7/7360.454	493.1/7436.825	670.0/7585.827	513.5/7451.000	692.3/7595.029
$\omega^4(\rho)$	0.5	510.7/7367.604	492.4/7443.115	670.1/7579.171	511.9/7453.013	688.7/7575.356
	0.6	519.8/7391.090	489.3/7452.531	664.7/7576.530	508.1/7465.410	677.1/7592.963
	0.7	545.2/7420.917	488.6/7459.393	659.2/7573.059	506.7/7478.322	679.3/7569.343
	0.8	595.3/7447.010	495.4/7471.887	653.2/7561.762	514.2/7481.435	663.8/7598.104
	0.9	707.5/7508.623	514.2/7495.982	637.5/7573.198	524.4/7505.240	648.0/7567.485
	1.0	939.2/7569.863	935.4/7579.394	1518.0/7429.612	930.9/7586.929	1515.7/7430.132

Table 8: Comparison of the matching approaches for the 500 vehicle instances, averaged over buffer settings $\delta = 1, ..., 6$.

Table 9: Comparison of the matching approaches for the 600 vehicle instances, averaged over buffer settings $\delta = 1, ..., 6$.

ω	ρ	GREEDY	$\mathrm{WSC}_{\mathrm{greedy}}^{\mathrm{LP}}$	$\mathrm{WSC}_\mathrm{RR}^\mathrm{LP}$	$\mathrm{WSP}_{\mathrm{greedy}}^{\mathrm{LP}}$	$\mathrm{WSP}_\mathrm{RR}^\mathrm{LP}$
ω^1		474.0/8792.016	643.0/9042.051	802.4/9057.959	609.9/9001.867	728.6/9034.953
	0.0	187.1/8358.231	171.4/8255.163	343.0/8630.545	179.8/8308.890	359.7/8637.470
	0.1	187.4/8349.654	168.3/8252.061	341.7/8633.042	178.1/8296.004	349.5/8648.709
	0.2	187.6/8346.485	165.1/8259.377	339.5/8620.907	175.4/8299.075	347.1/8648.782
	0.3	187.9/8365.431	163.4/8262.530	331.8/8639.139	173.4/8311.725	341.1/8650.158
	0.4	190.0/8376.003	161.4/8258.889	327.7/8608.274	170.5/8301.769	337.3/8658.075
$\omega^4(ho)$	0.5	196.2/8384.167	160.9/8261.204	324.6/8625.020	169.5/8298.686	335.1/8642.884
	0.6	200.7/8423.686	158.7/8269.980	322.3/8587.326	167.0/8308.498	325.8/8631.863
	0.7	213.5/8474.981	157.9/8273.689	313.7/8600.174	165.1/8328.017	320.5/8646.648
	0.8	246.8/8527.777	157.3/8321.315	305.5/8588.429	167.7/8343.331	311.0/8627.828
	0.9	323.0/8681.856	166.5/8379.565	287.8/8599.603	171.7/8399.199	290.0/8622.919
	1.0	520.0/8782.141	467.8/8800.594	1117.9/8783.759	464.5/8810.612	1115.1/8785.587

Table 10: Comparison of the matching approaches for the 700 vehicle instances, averaged over buffer settings $\delta = 1, .., 6$.

ω	ρ	GREEDY	$\mathrm{WSC}_{\mathrm{greedy}}^{\mathrm{LP}}$	$\mathrm{WSC}_\mathrm{RR}^\mathrm{LP}$	$\mathrm{WSP}_{\mathrm{greedy}}^{\mathrm{LP}}$	WSP_{RR}^{LP}
ω^1		156.2/9658.758	317.5/9999.219	479.7/10143.806	274.4/9973.591	406.9/10126.834
	0.0	58.8/8745.881	46.1/8458.867	159.2/9227.587	48.9/8519.776	157.2/9323.046
	0.1	59.3/8741.495	45.7/8448.529	158.9/9218.709	48.8/8505.500	150.6/9312.502
	0.2	59.2/8741.943	44.9/8443.738	156.8/9215.348	48.2/8496.357	150.7/9283.886
	0.3	59.0/8760.754	44.3/8442.400	151.6/9210.082	47.6/8495.190	145.2/9285.294
	0.4	59.5/8761.770	44.0/8430.118	145.9/9193.266	47.0/8484.818	143.4/9292.794
$\omega^4(ho)$	0.5	60.2/8792.172	43.0/8430.900	144.3/9201.860	45.6/8481.239	141.0/9261.781
	0.6	60.3/8846.850	41.9/8439.314	143.6/9167.630	43.5/8495.277	133.2/9262.294
	0.7	61.5/8947.845	40.3/8447.715	134.7/9170.654	42.4/8501.484	128.7/9268.173
	0.8	68.1/9123.182	38.9/8487.215	123.7/9177.005	40.5/8537.091	121.3/9227.817
	0.9	91.8/9485.880	39.2/8580.213	111.1/9138.746	40.6/8610.422	108.7/9180.898
	1.0	202.2/9879.019	151.2/9669.139	788.6/10049.986	149.0/9683.299	785.5/10056.420

(u)	0	GREEDY	WSC ^{LP}	WSC	WSP ^{LP}	WSP
	Ρ	GILLEDI	IT C C GREEDY		GREEDY	TIST RR
ω^1		15.7/9912.701	74.8/10720.931	281.1/10809.396	64.4/10634.283	211.2/10865.947
	0.0	15.3/8800.945	8.5/8449.284	114.0/9186.774	8.4/8510.451	96.7/9352.123
	0.1	15.3/8801.367	8.3/8438.259	111.6/9184.487	8.2/8501.105	90.0/9339.523
	0.2	15.2/8799.070	8.2/8428.931	110.8/9174.575	7.9/8490.644	89.5/9317.990
	0.3	15.3/8813.072	8.2/8424.803	109.1/9144.491	7.8/8486.144	86.1/9305.730
	0.4	15.2/8823.105	7.8/8417.973	103.9/9133.980	8.0/8474.138	85.4/9306.812
$\omega^4(ho)$	0.5	15.1/8853.028	7.6/8418.018	100.9/9145.333	7.4/8467.719	83.6/9274.370
	0.6	15.2/8900.045	7.4/8418.666	101.4/9104.079	6.8/8475.566	79.0/9259.791
	0.7	15.1/8994.387	7.1/8422.948	95.0/9087.233	7.0/8477.652	76.7/9250.806
	0.8	15.5/9190.426	6.4/8456.321	87.6/9074.320	6.5/8502.463	71.7/9202.167
	0.9	17.0/9613.796	7.3/8542.373	71.3/9069.061	7.3/8577.582	64.6/9135.124
	1.0	35.2/10432.105	16.8/9900.367	504.9/11323.463	17.8/9906.190	502.7/11328.454

Table 11: Comparison of the matching approaches for the 800 vehicle instances, averaged over buffer settings $\delta = 1, ..., 6$.

to identify promising parameter values for our resolution approach. We start with initial settings identified during development and preliminary experiments. Then, one by one, we vary the parameters in the following, arbitrary order: i) weights of the sorting criteria or the recreate operator, ii) acceptance criterion, iii) number of iterations in R&R (M_S and M_A), iv) average number of routes per decomposed problem, v) number of labels per balas-simonetti node (thickness value), vi) ruin parameters \bar{c} and L, vii) ruin parameters α and β , viii) blink rate in recreate operator, ix) perturbation relocate-exchange ratio, x) k-dSPP limits, xi) k-dSPP start time policy, and xii) recreate request limit. Tables 12–24 show the results of the tuning process steps.

Table 25 summarizes the final parameter settings after concluding the tuning.

Weights	Gap
6,2,1,2,2,2	0.163%/0.180%
6,2,1,2,2,4	0.104%/0.093%
6,2,1,2,4,2	0.000%/0.000%
6,2,1,4,2,2	-0.758%/-0.340%
6,2,4,2,2,2	-0.312%/-0.136%
6, 4, 1, 2, 2, 2	0.208%/0.152%
8,2,1,2,2,2	0.074%/0.062%

Table 12: Tuning results for the recreate sorting criteria weights.

Notes: Sorting criteria (random, far, close, pickup, start, deliveryend, time window length). We report the gap (unassigned requests / total distance) to the baseline setting (6,2,1,2,4,2).

Method	$T^{\rm init}$	Gap
Lin.R2R	0.111	1.063%/0.076%
Lin.R2R Lin.R2R	$0.222 \\ 0.333$	0.734%/0.141% 0.000%/0.000%
Lin.R2R	0.444	0.629%/0.425%
Exp.M Evp.M	$100 \\ 200$	7.681%/3.631%
Exp.M Exp.M	1000	6.228%/2.686%

Table 13: Tuning results for the acceptance criteria (Lin.R2R: Linear Record-to-Record; Exp.M: Exponential Metropolis).

Notes: We report the gap (unassigned requests / total distance) to the baseline setting (Lin.R2R 0.333).

Table 14: Tuning results for the number of iterations in the R&R (M_A) and for each sub-problem (M_S) .

M_A	M_S	Gap
$20e^4$	10000	1.183%/0.520%
$20e^4$	5000	0.434%/0.099%
$10e^4$	10000	1.228%/0.589%
$10e^4$	5000	0.000%/0.000%
$10e^4$	2500	0.284%/0.174%
$5e^4$	5000	1.123%/0.667%
$5e^4$	2500	-0.090%/0.078%
Notes:	We rep	ort the gap (unas-
signed	requests	/ total distance)
to the	baseline	e setting $(M_A =$
10e4, M	s = 5000).

Table 15: Tuning results for the average number of nodes per sub-problem (χ).

Range	Gap
[500, 500]	-2.008%/-1.444%
[500, 1000]	-1.229%/-0.850%
[1000, 1000]	0.000%/0.000%
[1500, 1500]	2.907%/1.687%

Notes: We report the gap (unassigned requests / total distance) to the baseline setting ([1000, 1000]).

Appendix C Fleet minimization

The classic PDPTW benchmark sets use a hierarchical objective function, prioritizing minimizing the fleet size over cost. We integrate a fleet minimization component to compare our approach to such sets and show its efficacy. We use a modified AGES, proposed by Sartori & Buriol (2020), outlined in Algorithm 3.

Thickness	Gap
4	-0.321%/-0.132%
8	0.000%/0.000%
12	0.046%/0.016%
16	0.107%/0.059%

Table 16: Tuning results for the number of labels maintained per node (thickness) in the Balas-Simonetti neighborhood.

Notes: We report the gap (unassigned requests / total distance) to the baseline setting (8).

Table 17: Tuning results for ruin parameters \bar{c} and L.

		L	
\overline{c}	5	10	20
5	-0.061%/-0.004%	-	-
10	0.046%/-0.081%	0.0%/0.0%	0.046%/-0.153%
15	- $0.015\%/-0.118\%$	-0.215%/-0.115%	0.614%/0.477%
20	—	-0.169%/-0.136%	—

Notes: We report the gap (unassigned requests / total distance) to the baseline setting ($\bar{c} = 10, L = 10$).

Table 18: Tuning results for ruin parameters $\alpha^{R\&R}$ and $\beta^{R\&R}$.

	$\alpha^{R\&R}$			
$\beta^{R\&R}$	0.25	0.50	0.75	
0.01	0.200%/0.119%	0.00%/0.00%	-0.046%/-0.032%	
0.10	—	0.277%/0.148%	- $0.384\%/-0.242\%$	
0.50	-	0.200%/0.110%	-0.031%/0.056%	

Notes: We report the gap (unassigned requests / total distance) to the baseline setting ($\alpha^{R\&R} = 0.50$, $\beta^{R\&R} = 0.01$).

Table 19: Tuning results for the blink rate in the recreate operator.

Blink	Gap
0.01	0.401%/0.387%
0.05	0.000%/0.000%
0.10	0.448%/0.258%

Notes: We report the gap (unassigned requests / total distance) to the baseline setting (0.05).

Factor	Gap
$1.00 \\ 1.66 \\ 2.00$	$\begin{array}{c} 0.401\%/0.218\%\\ 0.000\%/0.000\%\\ 0.448\%/0.296\%\end{array}$

Table 20: Tuning results for the number of perturbation moves in the ILS, relative to the number of requests in the instance.

Notes: We report the gap (unassigned requests / total distance) to the baseline setting (1.66).

Table 21: Tuning results for the ratio of relocate moves performed during permutation in the ILS.

Ratio	Gap
0.25	0.370%/0.214%
0.50	0.000%/0.000%
0.75	0.015%/0.054%

Notes: We report the gap (unassigned requests / total distance) to the baseline setting (0.50).

Table 22: Tuning results for KDSP distance and time limits when generating the auxiliary arc.

	Time [s]				
Dist. [m]	600	1200	1800	2400	
3000	_	_	0.525%/0.340%	_	
4000	33.812%/25.852%	0.046%/-0.153%	0.0%/0.0%	0.046%/0.057%	
5000	—	—	0.448%/0.197%	_	
6000	_	_	0.355%/0.215%	_	

Notes: We report the gap (unassigned requests / total distance) to the baseline setting (distance: 4000, time: 1800).

Table 20. Fulling results for the recombination methods	Table 23:	Tuning	results	for	the	recombination	methods.
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k-dSPP				
Earliest	Average Latest		Naïve	
-0.386%/-0.132%	0.0%/0.0%	0.525%/0.310%	7.623%/7.566%	

Notes: We report the gap (unassigned requests / total distance) to the baseline setting (average).

It uses ejection search Nagata & Kobayashi (2010), where an item in a list of unassigned requests is iteratively tested to be inserted into the current solution by potentially removing (ejecting) other requests if necessary. The ejection search is performed sequentially in the number of ejections considered. First, all ejections of a single request are tested. If no feasible insertion is possible, all

Limit	Gap
20	1.549%/0.852%
40	0.000%/0.000%
60	0.186%/0.078%
80	0.496%/0.258%

Table 24: Tuning results for the number of requests considered during recreate.

Notes: We report the gap (unassigned requests / total distance) to the baseline setting (40).

Parameter	Value	Parameter	Value
Acceptance criterion / T^{INIT}	Lin.R2R / 0.333	Ruin parameters	
ILS perturbation moves Z_A	$1.66 \cdot R $	\overline{c}	15
ILS relocate / exchange move ratio $\alpha^{\scriptscriptstyle \rm ILS}$	0.50	L	10
M_A, M_S	5000, 2500	$\alpha^{\text{R\&R}}$	0.75
Avg. number of nodes per sub-problem χ	500	$\beta^{R\&R}$	0.10
Balas Simonetti Nbh. thickness	4	Recreate parameters	
k-dSPP distance and time limits	4000, 1800	blink rate	0.05
k-dSPP start time policy	Earliest	sorting criteria weights	6,2,1,4,2,2
		request limit	40

ejections of two requests are tested. To guide the search, a penalty counter, ρ , of failed insertion attempts for each request is maintained during the search and used to prioritize ejections of potentially easy-to-reinsert requests (the fewest number of failed insertion attempts). The counter is maintained throughout multiple procedure calls and reinitialized in line 6 by applying a decay factor λ to each entry to nudge the search toward promising regions. If an insertion attempt fails, the penalty counter increases in line 15. When encountering ties, one possible ejection and insertion opportunity with minimal penalty is randomly selected using reservoir sampling. The sum of the failed insertion attempts is used to guide the search for two ejections. A perturbation procedure at line 17, right after attempting a potential ejection, aims to diversify the search for the next iteration by performing Z_F permutation moves. Herein, we either apply a relocate move with probability α^{AGES} or an exchange move otherwise. The search stops when a predefined number of perturbations M_F have been attempted. The corresponding tracker is reset if the stack size (the number of unassigned requests) decreases to a new minimum for the given fleet size.

Our main deviation from existing implementations is introducing the penalty decay factor λ . In our preliminary experiments, we observed tendencies of the guided search to reach similar prior penalty values before identifying more promising ejections. We avoid some of these time-consuming steps by not completely dismissing prior knowledge. However, we identified a larger factor necessary to prevent inverse effects on the search, where previously hard-to-insert requests need to be considered early to find improvements. Algorithm 3: Adaptive guided ejection search

Input: Feasible solution σ ; Maximum perturbation M_F ; Penalty counter ρ 1 while $i < M_F$ do $\vartheta \leftarrow \texttt{select_random_route}(\sigma);$ $\mathbf{2}$ $\sigma' \leftarrow \texttt{remove_route}(\sigma, \vartheta);$ 3 $E \leftarrow \texttt{initialize_stack}(\vartheta);$ 4 $min_{|E|} \leftarrow |E|;$ $\mathbf{5}$ $\rho[u] \leftarrow (1-\lambda)\rho[u]$, for every request u; 6 while $E \neq \emptyset \lor i < M_F$ do 7 $u \leftarrow \texttt{remove_request}(E);$ 8 if there is a feasible insertion of u in σ' then 9 $\sigma' \leftarrow \texttt{insert_request}(u, \sigma');$ 10 if $|E| < min_{|E|}$ then 11 $min_{|E|} \leftarrow |E|;$ 12 $i \leftarrow 0;$ 13 else 14 $\rho[u] \leftarrow \rho[u] + 1;$ 15 $\sigma' \leftarrow \texttt{eject_and_insert}(u, \sigma', E);$ 16 $\sigma' \leftarrow \texttt{perturb}(\sigma', Z_F);$ 17 $i \leftarrow i + 1;$ 18 if $E = \emptyset$ then $\sigma \leftarrow \sigma'$; 19 20 return σ

We integrated the described fleet minimization component into our algorithm by running AGES at the beginning of each iteration of our ILS approach, defined in Algorithm 1, right before line 8. The penalty counter ρ is initialized once with a value of 1 for every request and passed to the procedure each iteration. Table 26 lists the parameters used in our experiments.

Table 26: Final parameters settings used in the comparative experiments for the PDPTW benchmark.

DB-ILS				AGES			
M_A	M_S	χ	Z_A	λ	M_F	Z_F	$\alpha^{\rm ages}$
100000	10000	500	$ R \cdot 1.66$	0.8	1000000	10	0.58

Appendix D Solution structure

With the new set of instances generated based on the NYC Taxi and Limousine Commission (New York City Taxi and Limousine Commission 2015) data (abbreviated as NYC in this section) and their intended large-scale nature, we set out to identify how good solutions tend to be structured to compare ourselves with the best-fitting benchmark. To analyze the structure of efficient solutions for instances from the literature, we use the best-known solution details from https://www.sintef.no/

projectweb/top/pdptw/ and https://github.com/cssartori/pdptw-instances. In Figures 16–18, we compare the average number of requests per route (Figure 16), the average number of interleaving requests, i.e., how many requests are served concurrently (Figure 17), and the average number of blocks present per routes (Figure 18).

Requests per route. We observe in Figure 16 that NYC solutions have a low count of requests per route, similar to the Sartori & Buriol (2020) benchmark. The solutions for the Li & Lim (2001) instances tend to have a slightly higher average but spread to up to 8 requests per route.

Interleaving width. Figure 17 shows a stark difference in solution structure between Li & Lim (2001), with many concurrently served requests, and the others, which tend to be similarly low. One reason for this difference lies in the capacity restrictions present. The NYC instances allow up to 3 requests. Sartori & Buriol (2020) instances are generated in two ways: For the New York City instances, they consider passenger transportation with a request demand of up to 6 persons, which is also used as the maximum capacity of vehicles. For the other cities, they consider maximum capacity as demand per request. This leads to an average of 4 to 5.45 requests per vehicle, also reflected in the figure. Li & Lim (2001) instances are based on the classical Solomon instances for the VRPTW and do result in a wider spread with up to more than 30 concurrent requests served.

Blocks per route. Figure 18 shows that NYC solutions do consist of more blocks per route (sequences of visits where the vehicle is occupied) with lower buffer settings and gradually move to only two blocks with higher settings. Routes in Sartori & Buriol (2020) solutions tend to be composed of more blocks (up to more than 6), similar to lower buffer solutions. Li & Lim (2001) solution routes do average below two blocks, which implies that some vehicles may be occupied from the first visit to a pickup, to their last visit to a delivery.

Summary. We observe that solutions of our NYC instances share more similarities with the Sartori & Buriol (2020) benchmark set. This informed our decision to focus our design, tuning, and comparison efforts on these instances in our work.













