

A Universal Scheme for Dynamic Partitioned Shortest Path Index

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ABSTRACT

Shortest path (SP) computation is the fundamental operation in various networks such as urban networks, logistic networks, communication networks, social networks, etc. With the development of technology and societal expansions, those networks tend to be massive. This, in turn, causes deteriorated performance of SP computation, and graph partitioning is commonly leveraged to scale up the SP algorithms. However, the partitioned shortest path (PSP) index has never been systematically investigated and theoretically analyzed, and there is a lack of experimental comparison among different PSP indexes. Moreover, few studies have explored PSP index maintenance in dynamic networks. Therefore, in this paper, we systematically analyze the dynamic PSP index by proposing a *universal scheme* for it. Specifically, we first propose two novel partitioned shortest path strategies (*No-boundary* and *Post-boundary* strategies) to improve the performance of PSP indexes and design the corresponding index maintenance approaches to deal with dynamic scenarios. Then we categorize the partition methods from the perspective of partition structure to facilitate the selection of partition methods in the PSP index. Furthermore, we propose a universal scheme for designing the PSP index by coupling its three dimensions (i.e. *PSP strategy*, *partition structure*, and *SP algorithm*). Based on this scheme, we propose five new PSP indexes with prominent performance in either query or update efficiency. Lastly, extensive experiments are implemented to demonstrate the effectiveness of the proposed PSP scheme, with valuable guidance provided on the PSP index design.

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1 INTRODUCTION

Shortest Path (SP) query in dynamic networks is an essential building block for various applications affecting daily lives. Given an origin-destination pair, it returns the minimum traveling time in a road network [5], the fastest connection in the web graph [8], or the most intimate relationships in the social network [39, 58, 80]. With the development of urban traffic systems and the evolution of online interactions platforms, real-life networks tend to be massive, which brings great challenges to the scalability of the state-of-the-art [46, 47, 83, 85–87] with either heavy memory or expensive search overheads. For example, *2-Hop Labeling* [15] has the fastest query efficiency but requires $O(nm^{1/2})$ space (for a graph with n vertices and m edges) to store the index. Complementary, the search-based methods like *Dijkstra's* are inefficient on query processing on large networks. Moreover, because the networks are dynamic in nature with evolving structures or edge weights, extra efforts are required to handle the updates for SP indexes.

Graph partitioning is often used to improve the scalability of SP algorithms, enabling more complicated problems like time-dependent [43, 44] and constraint path computation [50, 51, 73] to be efficiently handled in large networks. It decomposes a large network into several smaller ones such that the construction time can be improved by parallelization, and the index sizes and update workloads can be reduced. As shown in Figure 1, we can compare them briefly in terms of *index construction time*, *storage space*, *query time*, and *update time*. On one extreme are the *direct search* algorithms [20, 30, 45, 84] that require no index and can work in dynamic environments directly but are slow for query answering. Then the *partitioned search* adds various information to guide and reduce search space [6, 14, 57] to improve query processing. On the other extreme, *Contraction Hierarchy (CH)* [25] and *2-hop Labeling (HL)* [2, 15, 46, 59] have larger index sizes, longer construction and maintenance time but faster query performance. Their partitioned versions [17, 22, 43, 47, 50–52, 75] are faster to construct with smaller sizes but longer query times. As these works have better performance than their unpartitioned versions, they create the following misconceptions regarding the benefits of partitions on pathfinding-related problems: 1) Graph partitioning and shortest path algorithms are irrelevant since they are two different research branches; 2) Different PSP indexes are irrelevant and very different from each other; 3) Applying graph partitioning is always better.

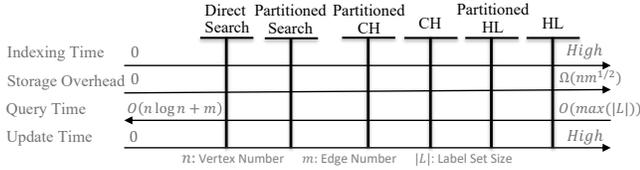


Figure 1: Performance Comparison of SP Algorithms

Motivations. These misconceptions are not always true. Although the partitioned shortest path (PSP) indexes have been widely applied in the past decade, to the best of our knowledge, they have not been studied systematically. Specifically, the existing solutions typically pick one partition method without discriminating their characteristics and figure out a way to make it work, and then claim benefits/superiority. Consequently, there lacks a generalized scheme to organize and compare the *PSP indexes* insightfully and fairly. Towards that, we postulate that both theoretical and experimental systematic study is needed for PSP indexes, so that a suitable index could be constructed for different scenarios.

Challenges. However, it is non-trivial to design such a scheme. Firstly, the current PSP indexes are delicately designed structures with tightly coupled components and all claim certain superiority. However, since no effort has been put into the abstract PSP index itself, it is unclear what determines their performance, when to use particular indexes, and how to design a suitable new PSP index for a new scenario. Therefore, we study the PSP index systematically in this work and identify three dimensions throughout the rich literature. Secondly, it is unclear how to construct, query, and update a new PSP index as all existing works only claim their unique structures can answer queries correctly and seldom involve index maintenance. Therefore, we dive into the *partitioned shortest path (PSP) strategy*, which determines how to coordinate multiple partitions for correct query answering, and summarizes the traditional PSP strategy. Additionally, we propose two novel *PSP strategies* together with their index update methods to improve the index performance. Besides, a *pruning-based overlay graph optimization* is also proposed to improve the index and update efficiency further. Thirdly, the graph partition methods are normally classified from different perspectives (like *vertex-cut / edge-cut, in-memory / streaming*, etc.), but these criteria are not path-oriented and thus hardly help to tell which partition method is suitable for certain path-oriented applications. Therefore, we propose a *path-oriented* partitioning classification from the perspective of partition structure for easier partition method selection. Lastly, after elaborating on each dimension of PSP index, we propose our universal PSP index scheme by entangling these dimensions. Based on this scheme, we propose five new PSP indexes to achieve better efficiency in either query or update operations by selecting the specific strategy or method on each dimension as per system requirements.

Contributions. Our contributions are listed as follows:

- We propose a universal PSP index scheme by decoupling the PSP index into three dimensions: *PSP strategy, partition structure, and SP algorithm*, such that the *PSP index* can be analyzed and designed systematically. (Section 2)
- We propose two novel PSP strategies called *No-Boundary* and *Post-Boundary* strategies to improve the index construction and maintenance efficiency over the traditional PSP strategy. We also

provide non-trivial correctness analysis for their index construction, query processing, and index maintenance. A novel *Pruning-based overlay graph optimization* is also designed to prune the unnecessary computation. (Section 3)

- We propose a new *path-oriented* partitioning classification from the perspective of partition structure for easier partition method selection in designing the PSP index. (Section 4)
- Based on the proposed PSP index scheme, we put forward a PSP index generator to create new PSP structures. Among them, we identify and design five new PSP indexes suitable for specific network structures and scenarios. (Section 5)
- We implement comprehensive experimental studies to explore the performance of PSP indexes under our proposed universal scheme and provide guidelines for designing PSP indexes with different application scenarios. (Section 6)

2 PRELIMINARY AND PSP DIMENSIONS

In this paper, we focus on the dynamic weighted network $G(V, E, W)$ with V denoting the vertex set, E denoting the edge set and $W : E \rightarrow \mathbf{R}^+$ assigning a non-negative weight to each edge – i.e. $w(u, v) \in W, \forall (u, v) \in E$ – which can increase or decrease in an ad-hoc manner. We denote the number of vertices and edges as $n = |V|$ and $m = |E|$. For each $v \in V$, we represent its neighbors as $N(v) = \{u, |(u, v) \in E\}$, u 's commonly called adjacent to v . We associate each vertex $v \in V$ with order $r(v)$ indicating its importance in G . In this paper, we use *boundary-first ordering* [51] to generate the order for the PSP indexes. For non-partitioned SP indexes, we leverage *degree* to decide the order of small-world networks while the *minimum degree elimination*-based ordering [7] for the road networks. A path $p = \langle v_0, v_1, \dots, v_k \rangle ((v_i, v_{i+1}) \in E, 0 \leq i < k)$ is a sequence of adjacent vertices with length of $l(p) = \sum_{i=0}^{k-1} w(v_i, v_{i+1})$. Given an origin s and destination t vertex (OD pair), the shortest path query $Q(s, t)$ calculates the path with the minimum length between them, i.e. $p_G(s, t)$ with the shortest distance $d_G(s, t)$. In the following, we introduce the PSP index dimensions and summarize each of them.

2.1 Dimensions of PSP Index

Based on a comprehensive literature review and analysis of the PSP index, we identify three critical dimensions of the PSP index: **1) Graph Partition Method**, which divides the graph into multiple subgraphs and is the key difference from the standalone SP indexes. However, the existing partition methods are not designed for path indexes, so how they affect the PSP index is unknown and deserves study; **2) PSP Strategy**, which provides a procedure to organize the index construction for multiple subgraphs and coordinate those indexes such that the query correctness is guaranteed. Only straightforward solutions exist currently as will be discussed in Section 2.3. However, this part is crucial because it determines the complexity of both the query and update; **3) SP Algorithm**, whose influence was thoroughly studied in our previous work [87]. With these three dimensions, we can form a space to position the existing methods into it. Given a massive dynamic network, these dimensions provide a roadmap to construct a PSP index by selecting a specific *graph partition method, SP algorithm* and *PSP strategy* such that the application requirement (query/update efficiency) can be satisfied. Next, we discuss and review these three dimensions.

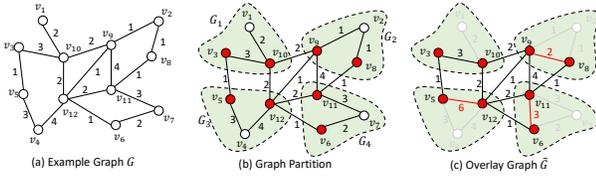


Figure 2: Example Graph G and Overlay Graph \tilde{G}

2.2 Graph Partitioning

DEFINITION 1 (GRAPH PARTITIONING). A graph G is decomposed into multiple disjoint subgraphs $\{G_i\}$ ($i = 1, 2, \dots, k$) with $V_i \cap V_j = \emptyset$ and $\bigcup V_i = V$. denoted as E_{inter} and E_{intra} .

We now discuss and categorize the vertex and edge in graph partitioning. $\forall v \in G_i$, we say v is a *boundary vertex* if there exists a neighbor of v in the another subgraph, that is $\exists u \in N(v), u \in G_j (i \neq j)$. Otherwise, v is an *inner vertex*. We represent the boundary vertex set of G_i as B_i and that of G as $B = \bigcup B_i$. For $(u, v) \in E$, we say (u, v) is an *inter-edge* if both its two endpoints u and v are boundary vertices from different subgraphs, i.e., $u \in B_i, v \in B_j, i \neq j$. Otherwise, it is an *intra-edge*. The corresponding edge sets are denoted as E_{inter} and E_{intra} , respectively. For example in Figure 2, the example graph is partitioned into four subgraphs G_1 to G_4 , with the red vertices denoting the boundaries ($B_1 = \{v_3, v_{10}\}$) and the white vertices representing the inner vertices. Edges outside partitions like (v_3, v_5) are inter-edges, and (v_3, v_{10}) is an intra-edge. Besides, the *boundary vertex* plays an important role in the shortest path computation with its **cut property**: given a shortest path $p(s, t)$ with its endpoints in two different subgraphs $s \in G_i, t \notin G_i$, there is at least one vertex v which is in the boundary vertex set of G_i on the path, that is $\exists v \in p(s, t), v \in B_i$. Therefore, *boundary vertices* would determine the correctness and complexity of partitioned shortest path computation (discussed in the following sections). Various partition methods are involved in the existing PSP methods such as *METIS* [37] in *G-Tree* [91] and *SHARC* [6]; *PUNCH* [18] in *COLA* [73], *FHL* [50, 52], *FHL-Cube* [51] and *THop* [43, 44]; *Tree Decomposition-based Partition* [59] in *Core-Tree* [46, 47]; *multi-level partitioning* [35] in *G-tree* [91, 92], *SPAH* [35], *CPR* [17] and [13, 14, 31]. But, their selections are performed in an ad-hoc manner or consider the boundary number, partition balance, and spatial awareness, rather than the SP computation itself.

2.3 Traditional PSP Strategy

The PSP index L typically consists of two components: the *partition indexes* $\{L_i\}$ for each subgraph (partition) G_i , and the *overlay index* \tilde{L} for the overlay graph which is composed of the boundary vertices of all partitions, i.e., $L = \{L_i\} \cup \tilde{L}$. Regardless of the specific SP algorithm, almost all the existing PSP indexes [13, 14, 17, 22, 29, 31, 36, 43, 44, 47, 49–52, 66, 71, 73–75, 90–92] construct the PSP index and process the queries under the following procedure.

Index Construction. It first leverages a partition method to divide the graph G into multiple subgraphs $\{G_i\}$ and then builds the PSP index L by the following four steps. As illustrated in Figure 3, *Step 1* precomputes the global distance between all boundary vertex pairs $(b_{i1}, b_{i2}), (b_{i1}, b_{i2} \in B_i)$ for each partition G_i and insert shortcuts $e(b_{i1}, b_{i2}) = d_G(b_{i1}, b_{i2})$ into G_i to get G'_i . For instance, the global distance between boundary vertex pairs $(v_3, v_{10}) \in G_1$,

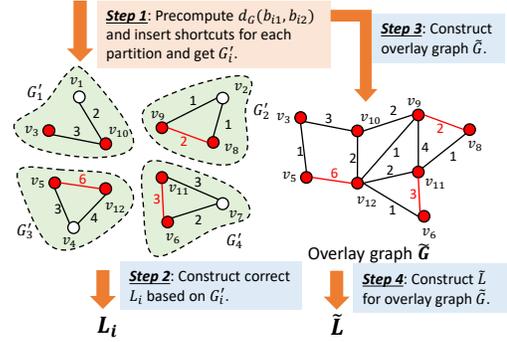


Figure 3: Traditional PSP Strategy Example

$(v_8, v_9) \in G_2, (v_5, v_{12}) \in G_3, (v_6, v_{11}) \in G_4$ are calculated and inserted into their corresponding subgraphs to form new subgraphs G'_1, G'_2, G'_3, G'_4 , respectively; *Step 2* constructs the SP index L_i based on G'_i . The index can be any type like *CH* [25], *PLL* [2] or *H2H* [59], and the index construction procedure among multiple partitions can be parallelized as it takes each partition as input independently; *Step 3* constructs the overlay graph \tilde{G} based on the precomputed shortcuts in *Step 1*. Specifically, \tilde{G} is composed of shortcuts between boundary vertex pairs $\{B_i \times B_i\}$ $((v_3, v_{10}), (v_8, v_9), (v_5, v_{12}), (v_6, v_{11}))$ and the inter-edge set E_{inter} $((v_3, v_5), (v_{10}, v_{12}), (v_9, v_{10}), (v_9, v_{12}), (v_9, v_{11}), (v_8, v_{11}), (v_{12}, v_{11}), (v_{12}, v_6))$, that is $V_{\tilde{G}} = \{B_i\}, E_{\tilde{G}} = \{B_i \times B_i\} \cup E_{inter}$; *Step 4* constructs the SP index \tilde{L} on \tilde{G} . Similar to L_i, \tilde{L} can be any index type.

Note that the construction of \tilde{L} (*Step 3, Step 4*) can be parallelized with L_i (*Step 2*) since they are independent and both rely on *Step 1*. All pairs of boundaries are independent of each other so we only need $|B|$ times of *Dijkstra's* with time complexity $O(n \log n + m)$. Then each partition's label L_i can be constructed in parallel, and \tilde{L} 's construction is also independent to them, so its complexity is the worst case of them: $\max\{O_C(G_i), O_C(\tilde{G})\}$, where O_C is the complexity of underlying index's construction time as our discussion is not fixed to any specific index type.

Query Processing. By referring to the *cut property* of boundary vertex, the shortest path queries are divided into two categories and processed by utilizing L :

Case 1: *Same-Partition* i.e., $\forall s, t \in G_i, Q(s, t) = d_{L_i}(s, t)$;

Case 2: *Cross-Partition*, i.e., $\forall s \in G_i, t \in G_j (i \neq j), Q(s, t) =$

$$\begin{cases} d_L(s, t) & s, t \in B \\ \min_{b_q \in B_j} \{d_L(s, b_q) + d_{L_j}(b_q, t)\} & s \in B, t \notin B \\ \min_{b_p \in B_i} \{d_{L_i}(s, b_p) + d_L(b_p, t)\} & s \notin B, t \in B \\ \min_{b_p \in B_i, b_q \in B_j} \{d_{L_i}(s, b_p) + d_L(b_p, b_q) + d_{L_j}(b_q, t)\} & s \notin B, t \notin B \end{cases}$$

In summary, when s and t are in the same partition, we can use L_i to answer $d_G(s, t)$; otherwise, we have to use L_i, L_j and \tilde{L} . The same-partition query complexity is $O_q(L_i)$, where O_q is the index's query complexity. The cross-partition query is made up of three parallel query sets, and the complexity is the worst of them: $\max\{B_i \times O_q(L_i), B_i \times B_j \times O_q(\tilde{L}), B_j \times O_q(L_j)\}$. The proof of the query processing can be found in the Appendix of our extended version [88].

2.4 Shortest Path Algorithms

We summarize the shortest path algorithms as follows: 1) *Direct Search* such as *Dijkstra's* [20] and *A** [30] searches the graph when the index cannot be constructed due to the large graph size (*QbS* [75], *ParDist* [14]) or complicated problem (*COLA* [73]). It takes no time in index update, but queries are slowest; 2) *Contraction Hierarchy (CH)* [25] is a widely used lightweight index that contracts the vertices in a pre-defined order and preserves the shortest path information by adding shortcuts among the contracted vertex's neighbors. The *search-based CH* [25] has a small index size but takes longer to build and maintain [26, 87] while *concatenation-based CH* [60, 77] is much faster to build and maintain if the tree-width is small. *CH's* query performance is generally around 10× faster than the *direct search* but much slower than the *hub labelings*. Surprisingly, no PSP index has used *CH* as underlying index; 3) *Pruned Landmark Labeling (PLL)*. Although many *hub labeling* methods have been proposed in the past decade, only two are widely used. Built by either *pruned search* [2] or *propagation* [34, 46, 85, 86], *PLL* is the only index that can work on the graph with large treewidth. Therefore, graphs with this property use it as the underlying index (e.g., *QsB* [22, 75] for its landmarks, *Core-Tree (CT)* [47, 90] for its core); 4) *Tree Decomposition (TD)* [11, 12, 59, 76, 83]. As another widely used hop labeling, it is much faster than *PLL* for graphs with smaller treewidth but cannot scale to large ones. For instance, *CT* [47, 90] uses it for its periphery, and *FHL* [50, 51] use it as a forest; 5) *All Pair Pre-computation* pre-computes all-pair distance with the fastest query, but incurs long index construction and large index storage. *G-Tree* [91, 92] and *ROAD* [41, 42] store the distance between boundary vertices in each partition.

3 NOVEL PSP STRATEGIES

As stated in Section 2, the traditional PSP strategy can be time-consuming for large networks since it requires a laborious computation of all-pair boundary distances using Dijkstra's search. To tackle this problem, we propose two novel PSP strategies and analyze how they could be leveraged to correctly answer the shortest path queries. Besides, we design the index update approaches systematically to support the dynamic application of both the existing and novel PSP strategies. Lastly, we proposed a pruning technique to slim the overlay graph to achieve better index performance. It should be noted that the PSP strategy is a general procedure that any *PSP index* needs to follow and can work with any type of *graph partition method* or *SP algorithm*. So we introduce the PSP strategies without mentioning the specific partitioning and SP approach.

3.1 No-Boundary and Post-Boundary Strategy

Since the traditional partitioned index starts by precomputing the all-pair distance among boundary vertices, we call this approach *Pre-Boundary Strategy*. It seems that this strategy, with the theoretical guarantee for correct partitioned shortest-path processing, could handle all the partitioned queries well. Nevertheless, the first step of the *Pre-Boundary* could be very time-consuming because only the index-free SP algorithms like *Dijkstra's* [20] or *A** [30]

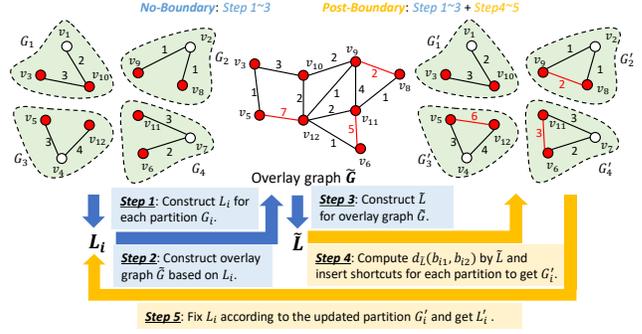


Figure 4: No-Boundary and Post-Boundary Strategies

could be utilized. As a result, the index construction and maintenance efficiency suffer when the graph has numerous boundary vertex pairs. However, it appears that pre-computing the all-pair boundary distance is an essential procedure for constructing the “correct” *PSP index*, otherwise the correctness of the subgraph index L_i cannot be guaranteed because the shortest distance between boundary vertices within one partition could pass through another partition. For instance, as shown in Figure 3, $p(v_6, v_{11}) = \langle v_6, v_{12}, v_{11} \rangle$ goes outside G_4 and passes through $v_{12} \in G_3$. As a result, $d(v_6, v_{11})$ cannot be answered correctly only with G_3 and the global shortest distance $d(v_6, v_{11})$ calculation is essential before constructing L_3 . Then a question naturally arises: do we really require precomputing the boundary all-pair distance first? In other words, does there exist a chance to keep index correctness by dropping this time-consuming step? Based on this brainstorm, we break the traditional misconception and propose a novel **No-Boundary Strategy** as follows, which significantly reduces the index construction/maintenance time by skipping the time-consuming pre-computation step.

Index Construction. It contains three steps as shown in Figure 4. With a graph partitioned into subgraphs $\{G_i\}$, we start by constructing the shortest path index L_i for each subgraph G_i parallelly in *Step 1*; Then *Step 2* construct the overlay graph \tilde{G} based on $\{L_i\}$. For instance, \tilde{G} is made up of boundary all-pair edges $(v_3, v_{10}), (v_8, v_9), (v_5, v_{12}), (v_6, v_{11})$ (with their weight calculated based on $L_1, L_2, L_3,$ and L_4) and inter-edges. It should be noted that although the overlay graph \tilde{G} has the same structure as that in the *Pre-boundary Strategy*, their edge weights differ as $\{L_i\}$ of them are constructed from different subgraphs; *Step 3* constructs the shortest path index \tilde{L} for \tilde{G} . Since the *partition indexes* $\{L_i\}$ are constructed in parallel first followed by the construction of \tilde{L} , the *No-Boundary* takes $\max\{O_c(G_i)\} + O_c(\tilde{G})$ time in index construction.

Query Processing. Now that L_i cannot answer G_i 's query correctly as the global distance between boundary all-pair is not covered, then how can *No-Boundary* answer query correctly? Before revealing it, we first prove that the correctness of \tilde{L} still holds even though it is built upon the incorrect $\{L_i\}$, that is the queries between boundary vertices can be correctly processed with \tilde{L} , using:

THEOREM 1. $\forall s, t \in B, d_G(s, t) = d_{\tilde{L}}(s, t)$.

PROOF. We divide all the scenarios into three cases as shown in Figure 5-(a): 1) $s, t \in G_i$ and $p_G(s, t)$ only passes through the interior of G_i , then it is obvious that $d_G(s, t) = d_{G_i}(s, t)$ holds. Since $\tilde{w}(s, t) = d_{G_i}(s, t)$ is leveraged for \tilde{G} 's construction, we can obtain that $d_G(s, t) = d_{\tilde{G}}(s, t) = d_{\tilde{L}}(s, t)$; 2) $s, t \in G_i$ with $p_G(s, t)$ going

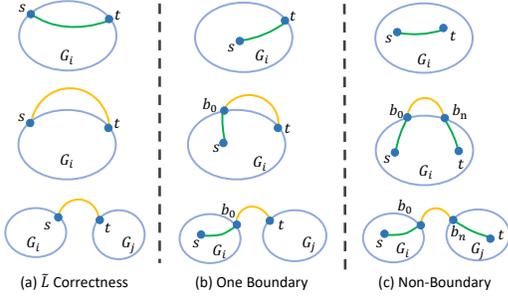


Figure 5: Different Categories of OD Distribution

outside of G_i ; 3) $s \in G_i$ and $t \in G_j$ ($i \neq j$). In the latter two cases, we take the concise form of $p_G(s, t)$ by extracting only the boundary vertices as $p_c = \langle s, b_0, b_1, \dots, b_n, t \rangle$ ($b_i \in B, 0 \leq i \leq n$). For two adjacent vertices $b_i, b_j \in p_c$, if b_i and b_j are in the same partition, then $\tilde{w}(b_i, b_j)$ can be correctly obtained in \tilde{G} via case 1); otherwise, (b_i, b_j) is an inter-edge with $\tilde{w}(b_i, b_j) = w(b_i, b_j)$ naturally correct. Thus, the shortest distance among boundary vertices can be correctly calculated by accumulating the edge weights on \tilde{G} . \square

Based on the above Theorem, we continue to answer the shortest distance queries by discussing all scenarios in two cases:

Case 1: Same Partition $s, t \in G_i$, we report $Q(s, t)$ as follows:

THEOREM 2. $\forall s, t \in G_i, d_G(s, t) = \min\{d_{L_i}(s, t), \min\{d_{L_i}(s, b_{i1}) + d_{\tilde{L}}(b_{i1}, b_{i2}) + d_{L_i}(b_{i2}, t)\}\}$, where $b_{i1}, b_{i2} \in B_i$.

PROOF. We denote $d_{L_i}(s, t)$ as d_2 , $\min\{d_{L_i}(s, b_{i1}) + d_{\tilde{L}}(b_{i1}, b_{i2}) + d_{L_i}(b_{i2}, t)\}$ as d_4 , and divide all the scenarios into two subcases:

Subcase 1: $p_G(s, t)$ does not go outside of G_i , as shown in the first row of Figure 5. No matter s and t are borders or not, $d_{L_i}(s, t)$ (i.e., d_2) is enough to answer $d_G(s, t)$ as L_i is built based on G_i which contains all necessary information for finding the shortest path.

Subcase 2: $p_G(s, t)$ passes outside of G_i , as shown in the second row of Figure 5. If s and t are both boundary vertices, $d_G(s, t) = d_{\tilde{L}}(s, t)$ holds by referring to Theorem 1. If s and t are both non-boundary vertices, we take the concise form of $p_G(s, t)$ by extracting only the boundary vertices as $p_c = \langle s, b_0, b_1, \dots, b_n, t \rangle$ ($b_i \in B, 0 \leq i \leq n$). Therefore, d_4 can correctly handle this case as the $d_G(b_0, b_n)$ can be answered by $d_{\tilde{L}}(b_0, b_n)$ as per Theorem 1, while $d_G(s, b_0)$ and $d_G(b_n, t)$ can be answered by $d_{L_i}(s, b_0)$ and $d_{L_i}(b_n, t)$ by referring to Case 1. If either s or t is a non-boundary vertex, its distance is the special case of d_4 and can be easily proved. \square

Case 2: Cross-Partitions, i.e., $s \in G_i, t \in G_j, i \neq j$, we process queries the same manner as that of *Pre-boundary Strategy*.

LEMMA 3. *The cross-partition queries can be correctly processed in the No-boundary Strategy.*

Note that the proof of Lemma 3 and all the other Lemmas of this paper are available in the Appendix of our extended version [88]. We can use \tilde{L} to answer $d_G(s, t)$ when s and t are both boundary vertices; otherwise, we have to use L_i, L_j and \tilde{L} . The intra-query complexity is $\max\{O_q(L_i), B_i \times B_j \times O_q(\tilde{L}), B_j \times O_q(L_j)\}$, while the inter-query takes $O_q(\tilde{L})$ time when s and t are both boundary vertices, and the complexity of other case is the worst of them: $\max\{B_i \times O_q(L_i), B_i \times B_j \times O_q(\tilde{L}), B_j \times O_q(L_j)\}$. Therefore, we make a bold attempt to enhance the performance of PSP index by skipping the heavy pre-computation, such that the index construction time

is largely reduced so as the index maintenance (as will introduced in Section 3.2). Surprisingly, with incorrect distance value in the *No-boundary index*, we prove theoretically that it can support the query answering with correctness guarantee.

By comparing the query processing procedure of the traditional *Pre-boundary* strategy and our proposed *No-boundary* strategy, we find that it is more complex for *No-boundary* to process those queries with endpoints in the same partition because of the incorrectness of $\{L_i\}$. We repair this weakness by fixing the incorrect boundary all-pair distance in each partition G_i and then transforming $\{L_i\}$ to their correct version. Specifically, we calculate the boundary all-pair distance $d(b_{i1}, b_{i2})$ within each partition G_i by leveraging \tilde{L} and insert its corresponding edge $e(b_{i1}, b_{i2})$ into G_i to get G'_i . For instance, $d(v_3, v_{10}), d(v_8, v_9), d(v_5, v_{12})$ and $d(v_6, v_{11})$ are calculated based on \tilde{L} and these edges (as colored red in Figure 4) are inserted into their corresponding partitions with new subgraphs $\{G'_i\}$ formed. Followed by, we treat those newly inserted edges as graph updates and refresh $\{L_i\}$ to $\{L'_i\}$ by invoking the index maintenance algorithms [22, 60, 83, 89] in each partition parallelly. As these procedures are implemented after the *No-boundary*, we call this strategy as *Post-boundary*. By referring to Theorem 1, the correct global distance information is contained in $\{G'_i\}$, so $\{L'_i\}$ can support the correct query processing of $Q(s, t)$ with $s, t \in G_i$. With the consistent query procedure as that of *Pre-boundary*, the query efficiency of *Post-boundary* reaches the state-of-the-art level.

3.2 Index Update for Different PSP Strategies

The partitioned shortest path has been widely used, however, how to maintain the index to support its application in dynamic environments has never been discussed. In this section, we propose the partitioned index update algorithms to guarantee the correctness of the partitioned index in dynamic scenarios.

Index Update for Pre-boundary Strategy. As shown in Figure 6-(a), when the weight of edge $e \in G$ changes, we first recalculate the boundary-all pair distance (as the step 1 in index construction) and identify the changed weight $e(b_{i1}, b_{i2})$ between boundary vertices in each partition G_i . Then we need to update the corresponding partition index L_i and overlay index \tilde{L} .

LEMMA 4. *The indexes of Pre-Boundary Strategy can be correctly updated with the above strategy.*

Step 1 takes $O_u(n \log n + m)$ time, while the partition and \tilde{G} index takes $\max\{O_u(G_i), O_u(\tilde{G})\}$ time to update in parallel, where O_u is the update complexity for different indexes.

Index Update for No-boundary Strategy. Since the weight change of inter-edges does not affect the index for each subgraph, we divide index updates into two scenarios as shown in Figure 6-(b). Scenario 1: Inter-edge weight change. When $e \in E_{inter}$ changes, only \tilde{L} needs an update; scenario 2: Intra-edge weight change. When $e \in E_{intra}$ ($e \in E_j$) changes, we first update L_j and compare the old and new weights of $e(b_{j1}, b_{j2})$ between boundary in G_j . If there is an edge weight update, we need to further update \tilde{L} . The update complexity is $\max\{O_u(G_i)\} + O_u(\tilde{G})$.

LEMMA 5. *The indexes of No-Boundary can be correctly maintained with the above update strategy.*

Index Update for Post-boundary Strategy. It is similar to *No-Boundary*, with an additional judgment and processing as shown in

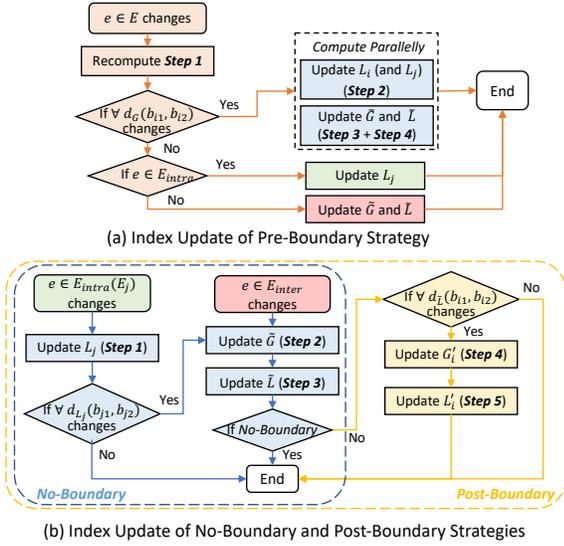


Figure 6: Index Update of Different PSP Strategies

Figure 6-(b). Scenario 1: Intra-edge weight change. Suppose $e \in E_i$ changes, we update G_i, L_i and then update \tilde{G}, \tilde{L} if any $d_{L_j}(b_{j1}, b_{j2})$ changes. After that we may need to further update $\{G'_i\}, \{L'_i\}$ if $d_{G'_i}(d_{i1}, d_{i2})$ and $d_{\tilde{L}}(d_{i1}, d_{i2})$ are different; scenario 2: Inter-edge weight change. Suppose $e \in E_{inter}$ changes, we update \tilde{G}, \tilde{L} and then update $\{G'_i\}, \{L'_i\}$. Therefore, the update time complexity is the linear combination of the above procedures: $\max\{O_u(G_i)\} + O_u(\tilde{G}) + O_q(\tilde{G}) + \max\{O_u(G_i)\}$.

LEMMA 6. *The indexes of Post-Boundary Strategy can be correctly maintained with the above update strategy.*

Remark 1. To construct the PSP index, *Pre-Boundary* precomputes the all-pair shortest distance among boundary vertices of each partition, and thus the newly added edges of G_i and \tilde{G} (red edges) are of correct edge weight. For example, the edge weight of $e(v_5, v_{12})$ is 6 which is the path length of the shortest path $p_G(v_5, v_{12}) = \langle v_5, v_3, v_{10}, v_{12} \rangle$. While *No-Boundary* skips the heavy precomputation and only leverage L_i to calculate boundary all-pair distance for overlay graph \tilde{G} construction. As a result, the edge weight $w_{\tilde{G}}(v_5, v_{12}) = 7$ is incorrect since it only preserves the local distance with corresponding local shortest path $p(v_5, v_{12}) = \langle v_5, v_4, v_{12} \rangle$ in G_3 . This innovation largely saves the index construction time while drags down the query answering speed. Then *Post-Boundary* makes up the query efficiency loss by inserting the correct shortest distance value between boundary pairs into each partition (e.g., insert $w(v_5, v_{12}) = 6$ to G'_3) and refresh $\{L_i\}$ to its correct version $\{L'_i\}$. In summary, there exists a balance of index performance among the three boundary strategies. In terms of the query processing, the *No-Boundary* is slower than *Pre-Boundary* and *Post-Boundary*, since the boundary vertices and path concatenation should be considered for *OD* within one partition. In terms of the index construction and update, the *No-Boundary* and *Post-Boundary* enjoy faster speed as the boundary all-pair distance computation can be neglected. In terms of the index size, the *Post-Boundary* needs twice the storage space as that of *No-Boundary* and *Pre-Boundary*, even though it has advantages in both query processing and index update. Therefore,

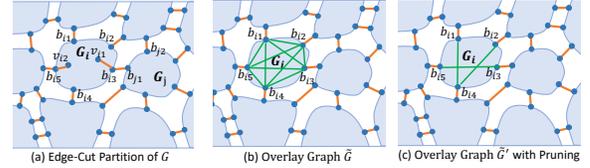


Figure 7: Example of Overlay Graph Simplification

no one strategy is better than the others in all aspects and could be selected based on the application scenario. Besides, it is worth noting that we illustrate PSP strategies in the form of *edge-cut* for clear presentation since the vertex-cut partitions can be easily converted to edge-cut partitions. Due to limited space, the conversion method is presented in extended version [88].

3.3 Pruning-based Overlay Graph Optimization

Though it is flexible to select the suitable boundary strategy according to application scenarios, we observe that all boundary strategies are hindered by the extremely dense overlay graph. The density of overlay graph \tilde{G} increases dramatically as the all-pair boundary vertices in each partition are connected during its construction, which badly affects the index performance by slowing down the index construction, query processing, and index update [87]. Then is it possible to improve index performance and decrease the density of \tilde{G} by deleting some unnecessary edges in $\{B_i \times B_i\}$? As proved in Theorem 1, the shortest distance between boundary vertices is well-preserved in \tilde{G} , then could the distance still be preserved in \tilde{G} if some edges are removed? We start exploring this question by categorizing the boundary vertices as followed:

DEFINITION 2 (HALF/FULL-CONNECTED BOUNDARY VERTEX). $b \in B_i$ is a half-connected boundary vertex if $\exists u \in N(b), u \in G_i, u \notin B_i$ and we denote them as B_i^H with $B^H = \{B_i^H\}$; otherwise, b is a full-connected boundary vertex and denoted as B_i^F with $B^F = \{B_i^F\}$.

As illustrated in Figure 7-(a), b_{13} is a half-connected boundary vertex since there exists $v_{11} \in N(b_{13}), v_{11} \notin B_i$; and b_{11}, b_{12}, b_{14} are full-connected boundary vertices. Initially, \tilde{G} can be viewed as composed of multiple complete graphs $\{B_i \times B_i\}$, which are connected by the inter-edges. Within this dense structure, the global distance between any two boundary vertices $s, t \in B$ is well-preserved. Let us consider a shortest path p with starting vertex $b_s \in B_i^F$ and target vertex $b_t \in B$, then the second vertex v_0 along p is a boundary vertex with $v_0 \in B, v_0 \in N(b_s)$ according to the definition above. Therefore, the global distance information of p will still be kept in \tilde{G} if we delete edges $\{(b_s, b_i)\}$ with $b_i \in B_i, b_i \notin N(b_s)$, since it is impossible for p to pass through these edges. The same principle holds for a shortest path ending with a fully connected boundary vertex $b_t \in B_i^F$ that its distance will also be preserved in \tilde{G} after deleting edges $\{(b_i, b_t)\}$ with $b_i \in B_i, b_i \notin N(b_t)$. Then for the shortest path starting and ending with the half-connected boundary vertices $b_s, b_t \in B_i^H$, it is necessary to insert an edge (b_s, b_t) into \tilde{G} to preserve its distance information, as the second vertex on this path could be a non-boundary vertex which is not contained in the overlay graph. As analyzed above, we can shrink $\tilde{G} (\{B_i \times B_i\} \cup E_{inter})$ to its slimmer version \tilde{G}' (composed of $\{B_i^H \times B_i^H\}, E_{inter}$ and B_i^F 's adjacent edges). Suppose $b_{11}, b_{12} \in N(b_{14})$ and let us only take G_i as an example for the clear presentation, the original overlay graph as

shown in Figure 7-(b) will be shrunk to a sparser one in Figure 7-(c) after the pruning technique. The following lemma demonstrates that the global distance can still be preserved in the overlay graph with the pruning technique.

LEMMA 7. $d_G(b_s, b_t) = d_{\tilde{G}}(b_s, b_t), \forall (b_s, b_t) \in B \times B$.

Therefore, we can enhance the index performance by pruning some edges in \tilde{G} and utilizing its sparser version \tilde{G}' , while preserving the global distance by referring to Lemma 7.

4 SP-ORIENTED PARTITION METHOD CLASSIFICATION

Having figured out how to achieve efficient query and update in partitioned graphs while guaranteeing the index correctness with the novel PSP strategies, we move on to the remaining two dimensions. As the *SP* index is well studied and analyzed in Section 2.4. Then, only the mist on the partition method remains: how to choose a partition method to achieve our preferred performance? This is not trivial because: 1) dozens of partition methods with different characteristics were proposed and applied in the past decades; 2) they are classified under different criteria [10], for example, from the perspectives of *partition manners* (spectral, flow, graph glowing, contraction and multi-level), *partition objectives* (balance and minimal cut), *computation manner* (in-memory, distributed and streaming), and *cut category* (edge-cut and vertex-cut), but it is not clear which criterion is beneficial to *SP* index; 3) their relationship with (or the influence on) *SP* index is unknown and has never been studied. Based on these doubts, we propose a novel SP-oriented classification of partition methods as follows:

1) **Planar Partition** treats partitions equally on one level. It decomposes a graph G into multiple equally-important subgraphs $\{G_i | 1 \leq i \leq k\}$ ($k \in [2, \infty]$) with 1) $\bigcup_{i \in [1, k]} V(G_i) = V, V(G_i) \cap V(G_j) = \emptyset$, or 2) $\bigcup_{i \in [1, k]} E(G_i) = E, E(G_i) \cap E(G_j) = \emptyset, \forall i, j \in [1, k], i \neq j$. Representative methods include *spectral partitioning* [4, 64], *graph growing* [19], *flow-based partitioning* [18, 69], *node-swapping* [24, 38], *multilevel graph partition (MGP)* [3, 37], etc..

2) **Core-Periphery Partition** ([9, 21, 54, 68]) treats the partitions discriminately by taking some important vertices as “Core” and the remaining ones as “Peripheries”. It decomposes a graph G into two distinct parts: a core subgraph G_c and $k - 1$ peripheral subgraphs $\{G_i | 1 \leq i \leq k - 1\}$ ($k \in [2, \infty]$), satisfying $\bigcup_{i \in [1, k-1]} V(G_i) \cup V(G_c) = V, V(G_c) \cap V(G_i) = \emptyset (\forall 1 \leq i \leq k - 1)$ and $V(G_i) \cap V(G_j) = \emptyset (\forall i, j \in [1, k - 1], i \neq j)$. There are two big streams depending on how the core is formed: *Core-Tree Decomposition* [47, 55, 90] forms the core through tree decomposition [67], and the resulting periphery part is a set of small-width trees. Specifically, it leverages *minimum degree elimination* [79] to contract vertices of lower degree first such that the graph is contracted from the edge towards the center, generating a set of growing trees (“Peripheries”) around a shrinking while denser graph (“Core”) formed by those non-contracted vertices. The contraction terminates when the width of one tree reaches the previously set threshold; *Sketch* [16, 22, 23, 29, 63, 65, 66, 72, 75] selects a set of vertices as *landmarks*, which can be regarded as the core and treats the remaining parts as periphery. It generally works on extremely large graphs where the above tree decomposition is impossible.

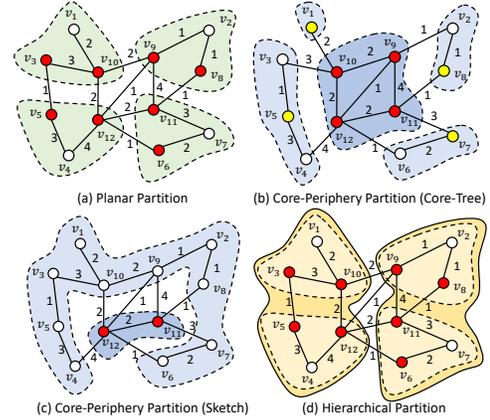


Figure 8: Different SP-Oriented Partitioning

3) **Hierarchical Partition** organizes the network partitions hierarchically and each level is equivalent to *planar partitioning*. It organizes a graph G hierarchically within H levels where each level h is a planar partitioning $\{G_i^h\}$ of G and $\exists G_i^{h-1} > G_i^h, \forall G_i^h (1 \leq h < H)$. Classic methods include *HiTi* [35], *SHARC* [6], *G-Tree* [92], *G*-Tree* [49], *SMOG* [31], and so on. A typical approach to create hierarchical partitions is by utilizing MGP, such as *METIS* [37] in *G-Tree* and *G*-Tree*.

Figure 8 shows the partition results of an example graph G with different partitioning methods, where the red vertices represent the corresponding boundary vertices. Specifically, *planar partitioning* generates four partitions with equivalent vertex size while *core-periphery partitioning* produces both “Core” and “Periphery” (the periphery of Core-Tree decomposition is a set of small-width trees while Sketch treats the non-core part as periphery. The yellow vertex in each periphery is the “root vertex”). *Hierarchical partitioning (HP)* organizes partitions hierarchically, and the leaf nodes (the lowest level partition result) may have the same partition result as planar partitioning if the same partitioning method is used. In addition, we summarize and categorize existing partition methods under our SP-oriented classification, as shown in Table 1.

Remark 2. As analyzed in Section 3, the boundary number of each partition will affect the partition method section as it is crucial to the computation complexity of *PSP index*. Specifically, the planar and hierarchical partitions have no limit to the boundary number, even though reducing the cut size is one of their optimization goals as the path index may suffer from large boundary numbers. Complementary, the core-tree decomposition limits the boundary number to be no larger than the pre-defined bandwidth such that their performance would not be deteriorated by the boundary if we set the bandwidth wisely. Therefore, the planar and hierarchical partitions are better used for small treewidth networks such that balanced partitions and fewer boundaries can be achieved together; while core-periphery is suitable for large treewidth networks as it deliberately limits the boundary vertex number by bandwidth.

5 UNIVERSAL PSP INDEX SCHEME AND NEWLY GENERATED PSP INDEXES

In this section, we propose a universal PSP index scheme and generator and put forward five representative PSP indexes for query-oriented or update-oriented applications based on this scheme.

Table 1: SP-Oriented Partition Methods Classification.

Partition Structure	Partition Method		Cut Category		Partition Objectives		Spatial-Aware
	Category	Representative Methods	Edge Cut	Vertex Cut	Balance	Minimal Cut	
Planar	Spectral Partitioning	SP [64], RSB [4]	✓	×	✓	✓	×
	Graph Growing	Bubble [19]	✓	×	×	✓	×
	Flow-Based Partitioning	PUNCH [18]	✓	×	✓	✓	✓
	Geometric Partitioning	RCB [33, 70]	✓	×	✓	✓	✓
	Node-Swapping Heuristic	KL [38], FM [24]	✓	×	✓	✓	×
	Minimum k -cut	k -cut [28]	✓	×	×	✓	×
	Multilevel Graph Partitioning	SCOTCH [61], METIS [37], KaHyPar [1]	✓	×	✓	✓	×
	Streaming Edge Partitioning	HDRF [56, 62], CLUGP [40]	×	✓	✓	✓	×
	In-Memory Edge Partitioning	NE [81], HEP [56]	×	✓	✓	✓	×
Core-Periphery	Core-Tree Decomposition	Core-Tree [21, 47, 55, 90]	✓	×	×	×	×
	Sketch	Sketch [16, 22, 23, 29, 63, 65, 66, 72, 75]	✓	×	×	×	×
Hierarchical	Edge-Cut Based HP	HiTi [35]	✓	×	×	×	✓
	Vertex-Separator Based HP	SHARC [6], G-Tree [92], G^* -Tree [49]	✓	×	✓	✓	×
		SMOG [31], ROAD [41, 42]	×	✓	×	×	×

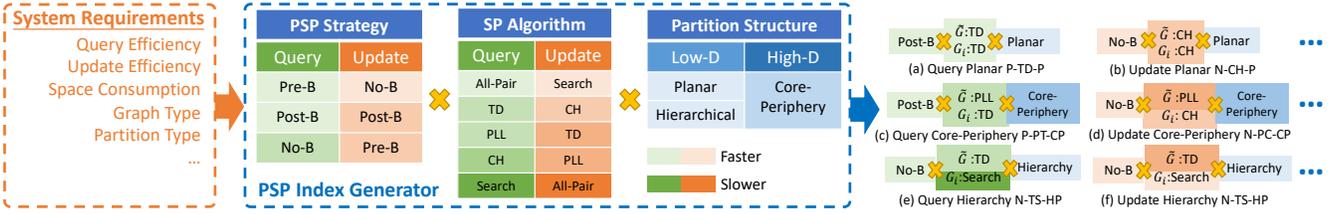


Figure 9: PSP Index Generator with PSP Index Scheme and Representative Generated PSP Indexes

5.1 PSP Index Scheme and Generator

After elaborating on each component of the PSP index, we now have our PSP index scheme ready as shown in Figure 9. The main part is the *PSP index generator*, which is formed by coupling those three components. For easy selection of the PSP strategy and SP algorithm, we sort all of their choices roughly in terms of query and update efficiency. As shown in Figure 9, the lighter color signifies faster efficiency for PSP strategy and SP algorithm. Meanwhile, planar and hierarchical partitions are more suitable for low-treewidth networks, while core-periphery can deal with high-treewidth graphs. Given the specific application requirements in terms of computation efficiency, space consumption, or partition type, our scheme could provide one PSP index with a specified PSP strategy, SP algorithm, and partition structure through the *PSP index generator*. For starters, the existing *PSP indexes* can find their positions in our scheme as will be discussed in Section 7, and we can compare them theoretically and fairly. Apart from this, we can further utilize it as a *PSP Index Generator* to create new PSP indexes that are suitable to user requirements, especially when all the existing ones are query-oriented, while there leaves a vast space of combinations for new ones. Specifically, the generated PSP index needs to choose the components from each of the three dimensions, and its name also has three parts corresponding to the three dimensions. Although we can enumerate them all, we choose to introduce the representative ones from the perspectives of *query*- and *update*-oriented under different partition structures, as shown in Figure 9.

5.2 Representative Generated New PSP Indexes

1. Query-Oriented Planar PSP Index: P-TD-P. Given a planar partition, the PSP strategies can apply directly to it. Next, we introduce how to construct the PSP indexes that are efficient in query processing under planar partition. Firstly in terms of PSP strategy, since both *Pre-Boundary* and *Post-Boundary* are efficient in

query answering and our proposed *Post-Boundary* is faster than *Pre-Boundary* in index construction, we use it as the PSP strategy. Secondly, in terms of the SP algorithm, we could choose *TD* as the index for both partitions and overlay graph. Although *all-pair* is faster, its space consumption is intolerable. Its structure is shown in Figure 9-(a). Specifically, the index construction takes $O(V_{max} \cdot (\log V_{max} + h_{max} \cdot w_{max}))$ time for the partition *TD*, $O(V_{\tilde{G}}(\log V_{\tilde{G}} + h_{\tilde{G}} \cdot w_{\tilde{G}}))$ for the overlay *TD*, $O(B_{max}^2 w_{\tilde{G}})$ for boundary correction, and $O(w_{\tilde{G}} + w_{max}^2 \cdot \delta)$ for the partition *TD* refresh, where δ is the affected shortcut number and *max* are the corresponding max values in the partitions. For the query processing, the intra-query takes $O(w_{max})$ as partition index is correct, and the inter-query takes $O(\max\{B_{max} w_{max}, B_{max}^2 w_{\tilde{G}}\})$ for the partition and overlay query and $O(B_{max}^2)$ for the combinations. For the index update, it takes $O(w_{max}^2 \delta)$ to update partition *TD*, $O(w_{\tilde{G}}^2 \delta)$ to update overlay *TD*, and $O(B_{max}^2 w_{\tilde{G}})$ to check boundaries.

2. Update-Oriented Planar PSP Index: N-CH-P. In terms of the PSP strategy, we use *No-Boundary* as it requires the least effort to update. In terms of SP algorithm, we can choose *CH* as the underlying index as it is fast to update while the query processing is better than direct search (Figure 9-(b)). Its index construction is faster with $O(V_{max} \cdot w_{max}^2 \cdot \log V_{max})$ for partition *CH* and $O(V_{\tilde{G}} \cdot w_{\tilde{G}}^2 \cdot \log V_{\tilde{G}})$ for overlay *CH*; the query time is longer with $O(\max\{B_{max} \cdot w_{max} \log V_{max}, B_{max}^2 \cdot w_{\tilde{G}} \log V_{\tilde{G}}\})$ for the intra- and overlay searching, and $O(B_{max}^2)$ for the combinations; the index update is faster with $O(\delta w_{max})$ for partition *CH* maintenance and $O(\delta w_{\tilde{G}})$ for overlay *CH* maintenance.

3. Query-Oriented Core-Tree PSP Index: P-PT-CP. The core-periphery partition index comprises the core index L_c and the periphery index $\{L_i\}$. It seems that L_c and L_i can be constructed by their corresponding subgraphs G_c and G_i , respectively. Although the core does not belong to any partition, they are connected to

the partitions, and we treat the core as the overlay graph. Different from the previous planar partition, the core here usually has a large degree so its index is limited to *PLL*. As for *sketch*, we omit it here because its *PLL* core + pruned direct search seems to be the only solution for huge networks. Next, we discuss the remaining parts. For the PSP strategy, *Post-Boundary* is utilized as the queries within the periphery can be handled without the core index. The periphery index uses *TD* because the periphery usually has a small degree. This structure is shown in Figure 9-(c). In terms of index construction, because the periphery is constructed through contraction, we regard it as a by-product of the partition phase and do not construct their labels in the first step. Then it takes $O(w_c E_c \log V_c + w_c^2 V_c \log^3 V_c)$ for the core *PLL*, $O(B_{max}^2 w_c \log V_c)$ for boundary correction, and $O(V_{max} \cdot w_{max}^2 \cdot \log V_{max})$ for periphery label. In terms of query processing, the intra-query takes $O(w_{max})$ time. The inter-query takes $O(\max\{B_{max} w_{max}, B_{max}^2 w_c \log V_c\})$ for the periphery and core, and $O(B_{max}^2)$ for the combinations. In terms of index update, it takes $O(w_{max}^2 \cdot \delta)$ for the periphery *TD*, $O(w_c E_c \log V_c)$ for the core *PLL*, and $O(B_{max}^2 w_c \log V_c)$ for boundary correction.

4. Update-Oriented Core-Tree PSP Index: N-PC-CP. As shown in Figure 9-(d), N-PC-CP uses *CH* for faster periphery update while *PLL* is for the core. It adopts *No-Boundary* strategy for faster update. In index construction, only core needs $O(w_c E_c \log V_c + w_c^2 V_c \log^3 V_c)$ time. The query time is longer with $O(\max\{B_{max} w_{max} \log V_{max}, B_{max}^2 w_c \log V_c\})$ for intra and core, and $O(B_{max}^2)$ for combinations. The index update is fast with $O(\delta w_{max})$ for periphery shortcuts and $O(w_c E_c \log V_c)$ for the core.

5. Update / Query-Oriented Hierarchical PSP Index: N-TS-HP. This category organizes L levels of partitions hierarchically with several lower partitions forming a larger partition on the higher level. We use L_i^l to denote the index of partition G_i^l on level l . Different from the SOTA *G-Tree* stream of indexes which uses all-pair in their hierarchical overlay graph, we replace it with the hierarchical labels for better query and update performance. Specifically, for vertices in each layer, we store their distance to vertices in their upper layers. As this is essentially 2-hop labeling, we use *TD* to implement it with orderings corresponding to the boundary vertex hierarchy. Such a replacement in the overlay index could answer queries and update much faster than the original dynamic programming-based layer all-pair index. As for the partitions, this structure tends to generate small partitions so we inherit the original search for fast query processing. Consequently, no partition index leads to inevitable boundary all-pair searches. Fortunately, our *No-Boundary* restricts the search space to the small partition compared with *G-Tree*'s whole graph *Pre-Boundary* (Figure 9-(e) and (f)). To construct the index, the partition boundary all-pair takes $O(B_{max} \cdot V_{max} (\log V_{max} + E_{max}))$, and its by-product boundary-to-partition can be cached for faster inter-query. Then the overlay *TD* takes $O(V_G (\log V_G + h_G w_G))$ time. In query processing, the intra-query takes $O(B_{max} V_{max} (\log V_{max} + E_{max}))$ for the direct search (very rare as the partitions are small), while the inter-query The query takes $O(B_{max} w_G)$ for intra- (constant time with cache) and hierarchical query, and $O(B_{max}^2)$ for combination. As for index update, it takes $O(B_{max} \cdot V_{max} (\log V_{max} + E_{max}))$ to update the partition all-pairs and $O(w_G^2 \cdot \delta)$ to update the overlay graph.

Table 2: Dataset Description

Type	Name	Dataset	V	E
Road Network	NY ¹	New York City	264,346	730,100
	FL ¹	Florida	1,070,376	2,687,902
	W ¹	Western USA	6,262,104	15,119,284
	US ¹	Full USA	23,947,347	57,708,624
Complex Network	GO ²	Google	855,802	8,582,704
	SK ³	Skitter	1,689,805	21,987,076
	WI ³	Wiki-pedia	3,333,272	200,923,676
	FR ²	com-Friendster	65,608,366	3,612,134,270

[1] <http://www.dis.uniroma1.it/challenge9/download.shtml> [2] <http://snap.stanford.edu/data> [3] <http://konect.cc/>

Table 3: Average Vertex Degree of Overlay Graph

Dataset	NY		FL		GO		SK	
	PUNCH	HEP	PUNCH	HEP	KaHyPar	HEP	KaHyPar	HEP
w/o Opt	48.68	325.98	36.17	719.25	947.36	9754.52	1420.18	34889.5
w/ Opt	48.68	320.61	36.17	714.30	912.24	7678.05	1409.98	27326.5

Remark 3. We note that these five PSP indexes are novel structures, each with unique technical challenges, especially in updating. Nevertheless, we believe the PSP scheme holds greater significance than these new indexes, as it can guide new PSP index designs. We omit their details due to limited space, but their source codes are provided [27]. Besides, we take the N-CH-P index as an example and elaborate on its details in the Appendix [?].

6 EXPERIMENTAL EVALUATION

In this section, we evaluate the effectiveness of the proposed methods and provide a systematic evaluation of PSP indexes. All the algorithms are implemented in C++ with full optimization on a server with 4 Xeon Gold 6248 2.6GHz CPUs (total 80 cores / 160 threads) and 1.5TB memory. The thread number is set to 150.

6.1 Experimental Settings

Datasets and Queries. We test on eight real-life datasets, including four weighted road networks (NY, FL, W, and US) and four complex networks (Web graph: GO and WI; Social network: SK and FR), as shown in Table 2. The edge weight of complex networks is generated by following the manner [85] that is inversely proportional to the highest degree of the endpoints. We randomly generate 10,000 queries and 1,000 update instances for each dataset to assess the query processing and index maintenance efficiency, respectively.

Partition Methods and Their Performance Metrics. To select the suitable partition method for PSP index, we implement 8 representative partition methods: *Bubble* [19], *RCB* [70], *KaHyPar* [1], *PUNCH* [18], *SCOTCH* [3], *METIS* [37], *HEP* [56], *CLUGP* [40], all of which are planar partition methods with *PUNCH* and *RCB* only working on road networks. Since core-periphery partition (including *Tree-Decomposition-based* partition [59] and landmark selection in *Sketch* [22]) and hierarchical partition (mostly using *METIS* [37]) methods are finite, they are selected by default in the corresponding PSP index. We evaluate the performance of a partition method from three aspects: overall border number $|B| = \sum_{i \in [1, k]} |B_i|$, average partition border number $\overline{|B_i|} = \frac{\sum_{i \in [1, k]} |B_i|}{k}$, and partition connectivity $R_C = \frac{\sum_{i \in [1, k]} |C(G_i)|}{k}$, where $C(G_i)$ is the connected component number of subgraph G_i .

PSP Indexes and Their Performance Metrics In terms of PSP indexes comparison, we evaluate 5 existing representative PSP indexes spreading over our proposed three partition structures: a)

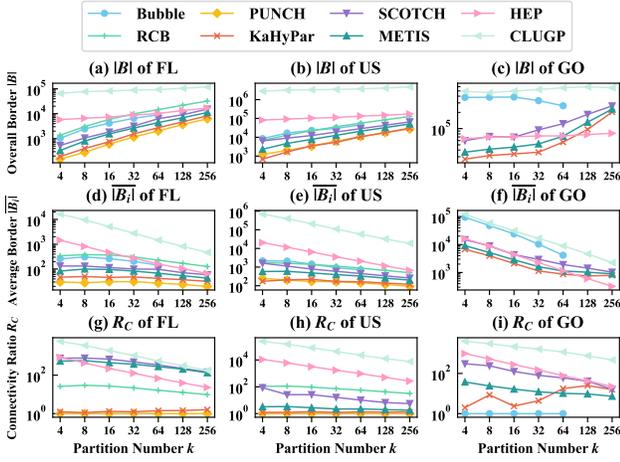


Figure 10: Partition Performance When Varying k .

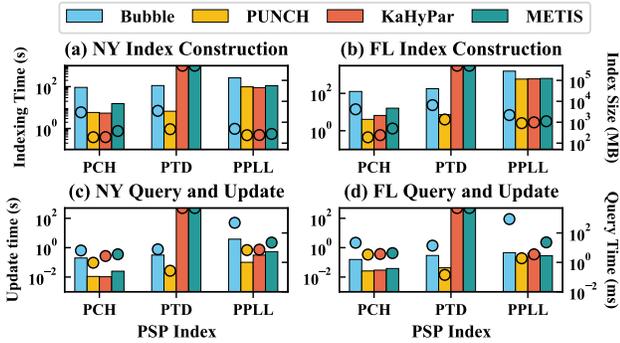


Figure 11: Effect of Partition Methods. Bar: Left, Ball: Right

FHL [50, 51] and *THop* [43] belongs to planar partition; b) *Core-Tree* (CT) [47, 90] and *Sketch* [22] belong to the core-periphery partition; c) *G-Tree* [91] belongs to the hierarchical partition. Besides, we implement our 5 newly proposed PSP indexes (see Section 5): *P-TD-P*, *N-CH-P*, *P-PT-CP*, *N-PC-CP*, and *N-TS-HP*. For the state-of-the-art shortest path algorithms CH [25, 60], TD [59, 83], and PLL [2, 85], we implement their partitioned version (named *PCH*, *PTD*, and *PPLL* respectively) to test the performance of different PSP strategies and select the suitable partition number. We measure the performance of the PSP index from four aspects: index construction time t_c , query time t_q , update efficiency t_u , and index size s . Note that we do not report the result (or set it to ∞) if the running time of a partition method or PSP index exceeds 24 hours.

Parameter Setting. We set the default partition number as 32 for all planar partitions and the bandwidth of core-tree indexes as 40 as per experimental results. The landmark number of Sketch is set to 20 by referring to [22]. As for the hierarchical partition, we follow G-tree [49, 91] and set the fan-out f as 4 and the maximum leaf node size τ as 128 in *NY*, 256 in *FL*, 512 in *W* and *US*, respectively.

6.2 Experiment Result Analysis

Exp 1: Performance of Partition Methods When Varying k .

We first report the performance of different partition methods with varying partition number k (we only report results on FL, US, and GO due to limited space; the full results are in the full version [88]).

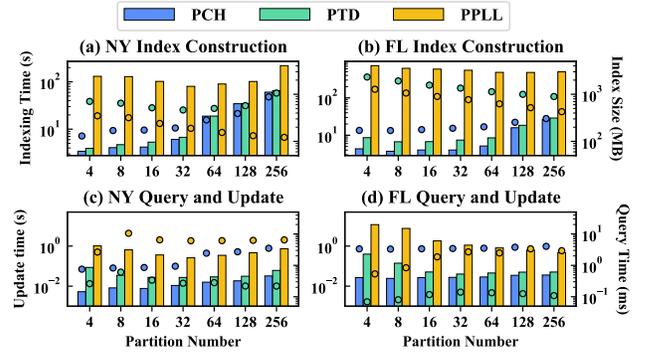


Figure 12: Effect of Partition Number. Bar: Left, Ball: Right

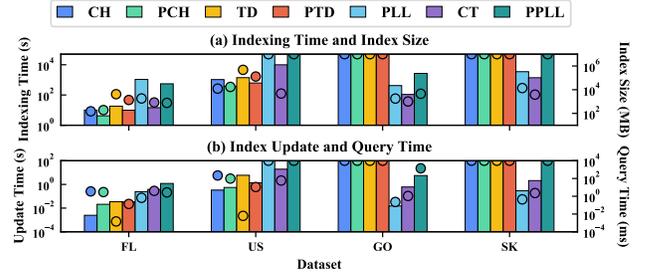


Figure 13: Comparison with Non-partitioned SP Indexes. Bar: Left, Ball: Right

As shown in Figure 10 (a)-(f), the overall border number $|B|$ generally increases, while the average border number $\overline{|B_i|}$ decreases as k grows from 4 to 256. This indicates that having a larger partition number can potentially speed up the query processing of the PSP index due to a smaller average partition border number. However, it also leads to a larger overlay graph, which can slow down the query processing and construction of the overlay index. Therefore, there should be a trade-off between them, and the best partition number typically varies for different PSP indexes. Furthermore, as per Figure 10 (g)-(i), only PUNCH and Bubble have a partition connectivity ratio R_C of 1, which implies that at least one partition generated by other partition methods is unconnected (with multiple separate components). Since PTD requires the vertices within one partition to be connected, only PUNCH and Bubble suit it.

Exp 2: Effect of Partition Method. Next, we further evaluate the effect of the partition method on the PSP indexes (PCH, PTD, and PPLL). In particular, we test PCH and PPLL with Bubble, PUNCH, KaHyPar, and METIS on road networks (*NY* and *FL*) while only testing PTD with PUNCH and Bubble by referring to Exp 1. Figure 11 shows that PUNCH and KaHyPar perform better than other partition methods in most cases due to smaller overall border number $|B|$ and average partition border number $\overline{|B_i|}$, which indicates that $|B|$ and $\overline{|B_i|}$ are important factors in the partition method selection. We select PUNCH as the default planar partition method due to its superior PSP index performance. It is worth noting that we also test PPLL with KaHyPar, METIS, and SCOTCH on complex networks. However, it fails to complete the index construction on the smallest graph GO within 48 hours due to the large and condensed overlay graph, which reveals that the PSP index with planar partition is generally unsuitable for complex networks.

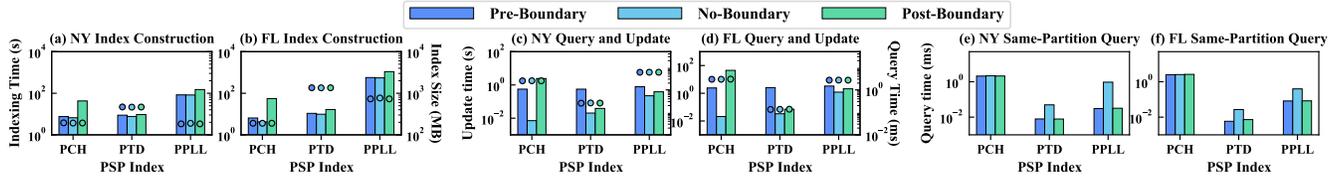


Figure 14: Effectiveness of Partitioned Shortest Path Strategy. (Bar: Left Axis; Ball: Right Axis)

Exp 3: Effect of Partition Number. As shown in Figure 12 (a)-(b), the indexing time of PCH and PTD increased as k increased from 4 to 256. This is because a larger k could lead to a larger overlay graph that cannot leverage thread parallelization, thus resulting in higher indexing time. By contrast, PPLL’s indexing time first decreases and then increases in NY while tending to decrease in FL because a larger k could also lead to a smaller $|B_i|$, thus reducing the overlay graph construction time. In Figure 12 (c)-(d), we observe that a smaller k generally improves the query time but leads to worse index update efficiency. Therefore, it is crucial to balance the query and update efficiency by selecting the appropriate partition number. We set the default value of k as 32 since it provided satisfactory performance in most cases.

Exp 4: Comparison with Non-partitioned SP Algorithms. We conduct a comparison between PSP indexes (including PCH, PTD, PPLL, and CT) and their non-partitioned counterparts (namely CH [25], TD [59], PLL [2]). As depicted in Figure 13, PSP indexes generally provide faster indexing time, smaller index size, and better scalability than their non-partitioned counterparts. For instance, PCH and PTD exhibit up to 2.76× and 2.25× faster index construction compared to CH and TD, respectively. Moreover, CT demonstrates 72.3× and 3.58× improvement over PLL in terms of indexing time and index size, respectively. However, PSP indexes may compromise query processing efficiency and index update time. It is worth noting that it also shows that planar structure is more suited for road networks, while core-periphery structure is better suited for complex networks, as PPLL does not outperform PLL in terms of indexing time and scalability.

Exp 5: Effectiveness of PSP Strategy. As shown in Figure 14 (a)-(d), the no-boundary strategy has the minimum index construction (about 14 – 35% speed-up) and index update time (up to two orders of magnitude speed-up compared with the pre-boundary strategy for PTD and PCH), which indicates the superiority of our no-boundary strategy. Suffering from the highest index construction time, the post-boundary strategy has the same query time as the pre-boundary strategy but a significant update efficiency improvement (14.6 – 36.7× speed up for PTD). To further demonstrate the effectiveness of our post-boundary strategy, we test the same-partition query efficiency as presented in Figure 14 (e)-(f). The post-boundary archives about 3.7 – 6.3× and 4.8 – 30.5× speed-up compared with the no-boundary strategy for PTD and PPLL, respectively. Such a significant performance gain comes from the post-boundary’s correct local partition index. Finally, we evaluate the effectiveness of pruning-based overlay graph optimization by measuring the average vertex degree of the overlay graph, as shown in Table 3. We test PUNCH and HEP on road networks while the KaHyPar and HEP on complex networks. The degree decreases with the pruning-based optimization in most cases. It cannot improve PUNCH since PUNCH already has a great partition result.

Exp 6: Comparison of All PSP Algorithms. We compare our proposed PSP indexes with the existing PSP indexes in Figure 15. We first analyze the performance of our proposed indexes against their counterparts: 1) P-TD-P has the same query performance and index size as FHL, but it is much faster to update with 11.1 – 202.7× speed-up. 2) N-CH-P has the fastest index construction and maintenance efficiency and the smallest index size among all planar PSP indexes, achieving up to 3.4×, 1004×, and 7.6× speed-up over FHL. 3) N-PC-CP has faster index construction and update than CT since it utilizes faster CH for tree index. P-PT-CP outperforms CT in terms of query processing as it is faster in same-partition queries. 4) N-TS-HP has faster index construction, smaller index size, faster query and update than G-Tree by leveraging TD for overlay index construction. 5) Sketch can scale up to very large graphs (the only algorithm that can be applied on graph FR), but its query efficiency is much slower than N-PC-CP and P-PT-CP due to the light index.

We next discuss the advantages of different partition structures. Figure 15-(b) shows that the state-of-the-art planar PSP indexes (N-CH-P, P-TD-P) and hierarchical PSP indexes (N-TS-HP) generally provide better query and index update efficiency for road networks than core-periphery PSP indexes. However, only core-periphery PSP indexes can be used on complex networks, which typically have large treewidth. Therefore, planar and hierarchical partitions are more suitable for road networks, while complex networks work better with core-periphery partitions.

6.3 PSP Application Guidance

Based on the experimental analysis in Section 6.2, we clarify the misconceptions mentioned in Section 1 and conclude the following observations: 1) all PSP indexes are essentially interrelated because they all consist of three dimensions in our PSP index scheme: graph partition method, PSP strategy, and shortest path algorithm. Moreover, we could combine the elements of these three dimensions to create a new PSP index; 2) it is important to choose the right partition structure for different types of graphs. Generally, for road networks, planar and hierarchical PSP indexes are more suitable than core-periphery PSP due to their efficient query and index update efficiency. On the other hand, core-periphery PSP indexes are more suitable for complex networks due to their excellent scalability; 3) although PSP indexes typically have faster indexing efficiency, smaller index size, and better scalability than their non-partitioned counterparts, they sacrifice query and index update efficiency.

According to our theoretical analysis and experimental results, we further provide our decision tree for PSP index design as illustrated in Figure 16. The first step in creating a new PSP index is to select a partition structure that is suitable for the specific application, such as graph type and size. For graphs with low treewidth, such as road networks, we recommend using planar or hierarchical

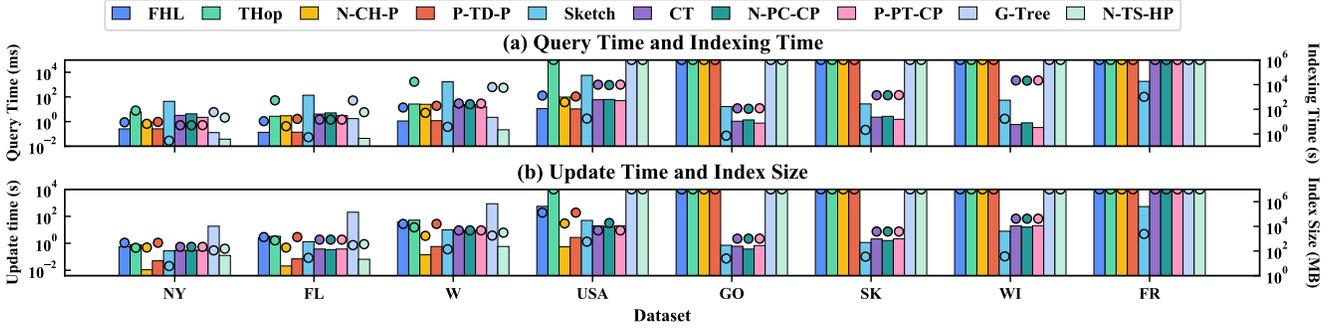


Figure 15: Comparison of All Shortest Path Indexes. Bar: Left, Ball: Right

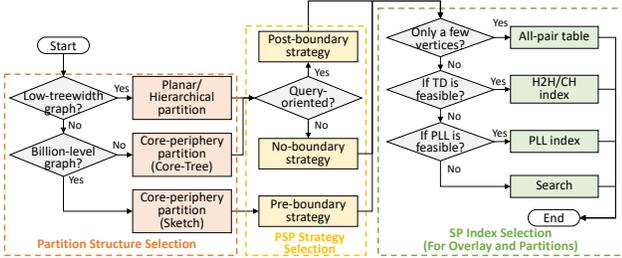


Figure 16: Our Decision Tree of PSP Index Design

partition structures. For larger graphs, we suggest using the core-periphery structure, with the core-tree structure for million-level graphs and sketch for billion-level graphs. The second step is to select the PSP strategy based on the required query processing and index update efficiency. For query-oriented applications, we recommend using the post-boundary strategy, while for update-oriented applications, the no-boundary strategy is more suitable. The third step is to choose the SP algorithm for the overlay and partitions. For a small number of vertices, we recommend using the most efficient all-pair table. For larger graphs, if tree decomposition is feasible, we suggest using TD or CH to achieve better query/update efficiency. If tree decomposition is infeasible, we recommend selecting PLL or even the direct search for SP algorithm selection.

7 RELATED WORK

Recently, multiple surveys about the SP computation have emerged, focusing on either specific graph types (like transportation networks [5] or complex networks [82]) or static graphs [48, 53, 78]. [87] investigates the computation in dynamic graphs, but it does not consider the partitioned counterpart. The partitioned pathfinding framework is proposed in [13, 14], however, the dynamic version is not discussed and they can be covered by the traditional PSP strategy. In this paper, we study the dynamic PSP problem systematically and explore its performance in both road networks and complex networks by involving the state-of-the-art discovery of SP algorithms and partition methods. Next, we discuss and categorize the existing PSP methods based on our scheme:

1) Pre-Boundary + Search/All-Pair + Hierarchy/Planar: These methods (*HiTi* [36], *Graph Separators*, *Customizable Route Planning* [17] [31], *ParDiSP* [13, 14]) pre-compute the shortest distance between boundaries (some hierarchically) to guide the search. In a broad sense, *Arc-flag* [57] and *SHARC* [6] also belong to this category.

G-tree [49, 91] uses dynamic programming to compute the distance between layer’s all-pair information to replace searching, and it is widely used in *kNN* [92], *ride-sharing* [71], *time-dependent routing* [74], and *machine learning-based path finding* [32]. They are slow to construct due to *Pre-B* and slower to query to direct search;

2) Pre-Boundary+Search/PLL+Planar: *COLA* [73] builds the labels for the *skyline shortest path* on the overlay graph to answer the constrained shortest path query. This structure’s construction suffers from *Pre-B* and query suffers from searching;

3) Pre-Boundary+TD/TD+Planar: *FHL* [50–52] partition builds the *TD* both within and between partitions for multi-dimensional skyline paths. As validated in Exp 4, our *Post-B* can speedup *Pre-B*’s construction dramatically;

4) Pre-Boundary+PLL/ PLL+Planar: *T2Hop* [43, 44] utilizes two layers of *PLL* to reduce the complexity of long range time-dependent paths, and this structure’s performance is limited by *PLL*;

5) No-Boundary+ PLL/TD+Core: *Core-Tree* [47, 90] is the baseline that we already discussed;

6) Pre-Boundary+Search/All-Pair/PLL+Sketch: This category works on huge graphs where index is nearly impossible so only a small number of landmarks are selected to either help prune the search [16, 22, 75] or approximate result [29, 66].

In sum, all the existing PSP methods have their limitation, warranting the comprehensive investigation embarked on in this paper.

8 CONCLUSIONS

In this work, we systematically study the partitioned shortest path index and propose a universal PSP scheme by concomitantly considering three dimensions: PSP strategy, SP algorithm, and partition structure. For the PSP strategy, we propose two new strategies and pruned-boundary optimization for better index performance. We also provide index maintenance solutions for both traditional and novel PSP strategies. For partition structure, we propose a new path-oriented partition method classification. Furthermore, we construct our PSP index scheme by seamlessly coupling those three dimensions. We also propose a PSP index generator and five new PSP indexes that are either more efficient in query or update than the current state-of-the-art. Systematic and extensive experimental evaluations fully validate our findings and prove the effectiveness of our new indexes guided by the insightful PSP index scheme. Finally, PSP index design guidelines are provided to help future researchers and practitioners design their indexes. Our future work will address recommendation aspects for such designs.

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9 APPENDIX

9.1 Query Correctness Proof of Traditional PSP Strategy

We formally prove the query correctness of the traditional PSP strategy (i.e., *Pre-boundary* strategy) in the following two lemmas:

LEMMA 8. *The same-partition query can be processed correctly.*

PROOF. We prove this by dividing all cases into 2 sub-cases:

Sub-Case 1: $p_G(s, t)$ passes through no vertex outside G_i , that is $\forall v \in p_G(s, t), v \in G_i$, then $d_G(s, t) = d_{G_i}(s, t) = d_{L_i}(s, t)$;

Sub-Case 2: $p_G(s, t)$ passes out of G_i ($\exists v \in p_G(s, t), v \in G_j, j \neq i$). Suppose the first and last vertex on $p_G(s, t)$ which are not in G_i is v_1 and v_2 ($v_1 \notin G_i, v_2 \notin G_i$, and v_1, v_2 could be the same vertex). Then we denote the vertex right before v_1 and right after v_2 along $p_G(s, t)$ are v_{b1} and v_{b2} . Both v_{b1} and v_{b2} are boundary vertices with $v_{b1}, v_{b2} \in B_i$. We can decompose the length of $p_G(s, t)$ as $d_G(s, t) = d_G(s, v_{b1}) + d_G(v_{b1}, v_{b2}) + d_G(v_{b2}, t)$, and $p_{G_i}(s, t)$ on G_i as $d_{G_i}(s, v_{b1}) + d_{G_i}(v_{b1}, v_{b2}) + d_{G_i}(v_{b2}, t)$. Since $d_G(s, v_{b1}) = d_{G_i}(s, v_{b1})$ and $d_G(v_{b2}, t) = d_{G_i}(v_{b2}, t)$ by referring to the above *sub-case 1*, and $d_G(v_{b1}, v_{b2}) = d_{G_i}(v_{b1}, v_{b2})$ as indicated in the *Step 1 of Index Construction*. So $d_G(s, t) = d_{G_i}(s, t) = d_{L_i}(s, t)$ holds. \square

LEMMA 9. *The cross-partition query can be processed correctly.*

PROOF. We discuss those 4 sub-cases one by one:

Sub-Case 1: Both s and t are boundary vertices. Suppose that the concise path of $p_G(s, t)$ by extracting the boundary vertices is $p = \langle s = b_0, b_1, \dots, b_l = t \rangle$ with $\forall b_h \in p(0 < h \leq l), b_h \in \tilde{G}$. Then $d_G(s, t) = \sum_{h=0}^{l-1} d_G(b_h, b_{h+1})$. If b_h and b_{h+1} are in the same partition, then $d_G(b_h, b_{h+1}) = e_{\tilde{G}}(b_h, b_{h+1})$ with $(b_h, b_{h+1}) \in \tilde{G}$ by referring to the *Index Construction*. If b_h and b_{h+1} are in different partitions, then $(b_h, b_{h+1}) \in E_{inter}$ and $d_G(b_h, b_{h+1}) = e_{\tilde{G}}(b_h, b_{h+1})$. So it can be proved that $d_G(s, t) = d_{\tilde{G}}(s, t) = d_{\tilde{L}}(s, t)$;

Sub-Case 2: s is a boundary vertex and t is an inner vertex of G_j . Suppose the last boundary vertex on $p_G(s, t)$ is b . Then $b \in B_j$ since $p_G(s, t)$ would reach t by entering G_j through any vertex in B_j . The shortest distance can be calculated as $d_G(s, t) = d_G(s, b) + d_G(b, t)$. We can get $d_G(s, b) = d_{\tilde{G}}(s, b)$ by referring to the *Sub-Case 1* and $d_G(b, t) = d_{\tilde{G}}(b, t)$ according to Lemma 8, such that $d_G(s, t) = d_{\tilde{G}}(s, b) + d_{G_j}(b, t) = d_{\tilde{L}}(s, b) + d_{L_j}(b, t)$;

Sub-Case 3: s is an inner vertex and t is a boundary vertex. This is the reverse version of *Sub-Case 2*;

Sub-Case 4: Neither s nor t is boundary vertex. It can be proved by extending *Sub-Case 2*. Therefore, both *sub-case 3* and *sub-case 4* can be proved similarly as *sub-case 2*. \square

9.2 Proof of Lemmas

Proof of Lemma 3.

PROOF. We prove and discuss it with three subcases as shown in Figure 5-(c).

Subcase 1: $s, t \in B$, then $d_G(s, t)$ can be correctly answered according to Theorem 1.

Subcase 2: $s \in B$ or $t \in B$. As shown in the second illustration in Figure 5-(c), suppose s is an inner vertex of G_i and t is a boundary vertex, we take the concise $p_{s,t}$ by extracting the boundary vertices as $p_c = \langle s, b_0, \dots, b_n, t \rangle$. Then $b_0 \in B_i$ and $p_{s,t}$ can be treated as

concatenated by two sub-paths $p_{s,b_0} \oplus p_{b_0,t}$. Specifically, $d_G(s, b_0) = d_{L_i}(s, b_0)$ by referring to the Subcase 1 of Case 1, and $d_G(b_0, t) = d_{\tilde{L}}(b_0, t)$ by referring to Theorem 1.

Subcase 3: $s \notin B, t \notin B$, which is the extended case of Subcase 2, so $d_G(s, t)$ can also be correctly answered. \square

Proof of Lemma 4.

PROOF. First of all, we need to recalculate *Step 1* to identify the affected edges $e(b_{i1}, b_{i2})$ between boundary vertex pairs in each partition. Even though this step is time-consuming, it cannot be skipped since it would be hard to identify the affected edges. For example, suppose the shortest path between the boundary pair (b_{j1}, b_{j2}) in G_j passes through an edge $e \in G_i$ with $d_{\tilde{G}}(b_{j1}, b_{j2}) = d_0$. When e increases, we could update L_i and then \tilde{L} . But \tilde{L} cannot be correctly updated since it could be that $d_{\tilde{G}}(b_{j1}, b_{j2}) > d_0$, such that $d_{\tilde{G}}(b_{j1}, b_{j2})$ cannot be refreshed to the correct value. It is because d_0 contains the old smaller edge weight while cannot be identified, since the shortest distance index always takes the smallest distance value. Then we could select those affected edges $e(b_{i1}, b_{i2})$ by comparing their old and new weights. Lastly, we update their corresponding partition index L_i and \tilde{L} in parallel. \square

Proof of Lemma 5.

PROOF. In the *inter-edge update* case, since $e \in \tilde{G}, e \notin G_i, \forall e \in E_{inter}$, the weight change of e could only affect the correctness of \tilde{L} . So only \tilde{L} should be checked and updated. In the *Intra-edge update* case, since $e \in G_i$, its weight change will firstly affect L_i . Then it could affect \tilde{G} as $e_{\tilde{G}}(b_{i1}, b_{i2}) = d_{L_i}(b_{i1}, b_{i2})$. So \tilde{L} also needs update if $e_{\tilde{G}}(b_{i1}, b_{i2})$ changes after checked. \square

Proof of Lemma 6.

PROOF. Firstly, we prove the necessity to keep both $\{G_i\}, \{L_i\}$ and $\{G'_i\}, \{L'_i\}$. Similarly to the *Pre-Boundary Strategy*, those boundary edges in $\{G'_i\}$ would keep the old smaller value such that the index could not be correctly updated as explained in Lemma 4. So keeping $\{G_i\}, \{L_i\}$ gives us a chance to update $\tilde{L}, \{L_i\}$ correctly as proved in Theorem 5. Then, following the *No-Boundary Strategy* update, we recompute the shortest distance between the all-pair boundaries leveraging \tilde{L} and compare their values on L'_i , then update G'_i, L'_i . \square

Proof of Lemma 7.

PROOF. We denote the concise form of $p_G(b_s, b_t)$ by only taking the boundary vertices as $\{b_s=b_0, \dots, b_j, \dots, b_t=b_h\}, (0 < j < h)$.

When $h=1$, if either $b_s \in B^F$ or $b_t \in B^F$, then $d_G(b_s, b_t) = w_G(b_s, b_t)$ holds with $b_s \in N(b_t)$ by referring to definition 2. Since $(b_s, b_t) \in \tilde{G}$, $d_G(b_s, b_t) = d_{\tilde{G}}(b_s, b_t)$ holds. The same applies when $b_s, b_t \in B^H$ with two endpoints in different subgraphs $b_s \in B_i, b_t \in B_j (i \neq j)$. If $b_s, b_t \in B^H$ with $b_s, b_t \in B_i$, edge (b_s, b_t) are inserted into \tilde{G}' with their global shortest distance in both *Pre-Boundary* and *Post-Boundary* strategies, so $d_G(b_s, b_t) = d_{\tilde{G}'}(b_s, b_t)$ naturally holds. In the *No-Boundary* strategy, though only the local shortest distance is inserted in the overlay graph with $d_{G_i}(b_s, b_t) = d_{\tilde{G}'}(b_s, b_t)$, it holds that $d_G(b_s, b_t) = d_{G_i}(b_s, b_t)$ since there exists no other boundary vertex besides two endpoints in $p_G(b_s, b_t)$ and it only pass through G_i , so $d_G(b_s, b_t) = d_{\tilde{G}'}(b_s, b_t)$ holds.

Table 4: Partitioned SP Strategies Comparison

	Operations	Procedure	Complexity	
Pre-Boundary	Construction	$\{B_i \times B_j\} \rightarrow \{L_i\} \rightarrow L$	$O(n \log n + m) + \max\{O_c(G_i), O_c(\tilde{G})\}$	
	Query	Intra	L_i	
		Inter	$L_i \oplus L \oplus L_j$	$\max\{B_i \times O_q(L_i), B_i \times B_j \times O_q(L), B_j \times O_q(L_j)\}$
Update	$\{B_i \times B_j\} \rightarrow \{L_i\} \cup L$	$O(n \log n + m) + \max\{O_u(G_i), O_u(\tilde{G})\}$		
Post-Boundary	Construction	$\{L_i\} \rightarrow L \rightarrow \{B_i \times B_j\} \rightarrow \{L_i\}$	$\max\{O_c(G_i)\} + O_c(\tilde{G}) + O_q(\tilde{G}) + \max\{O_u(G_i)\}$	
	Query	Intra	L_i	
		Inter	$L_i \oplus L \oplus L_j$	$\max\{B_i \times O_q(L_i), B_i \times B_j \times O_q(L), O_q(B_j \times L_j)\}$
Update	$\{L_i\} \rightarrow L \rightarrow \{B_i \times B_j\} \rightarrow \{L_i\}$	$\max\{O_u(G_i)\} + O_u(\tilde{G}) + O_q(\tilde{G}) + \max\{O_u(G_i)\}$		
No-Boundary	Construction	$\{L_i\} \rightarrow L$	$\max\{O_c(G_i)\} + O_c(\tilde{G})$	
	Query	Intra	$L_i \oplus L \oplus L_i$	$\max\{B_i \times O_q(L_i), B_i \times B_j \times O_q(L)\}$
		Inter	$L_i \oplus L \oplus L_j$	$\max\{B_i \times O_q(L_i), B_i \times B_j \times O_q(L), B_j \times O_q(L_j)\}$
Update	$\{L_i\} \rightarrow L$	$\max\{O_u(G_i)\} + O_u(\tilde{G})$		

Because different index structures have different complexities but sharing the same input size and logical procedure, we use the following notations to represent the logical complexities: O_c is the index construction complexity, O_q is the query complexity, and O_u is the index update complexity.

When $h > 1$, the shortest path distance is accumulated as $d_G(b_s, b_t) = \sum_{j=0}^{h-1} d_G(b_j, b_{j+1})$. Since $d_G(b_j, b_{j+1}) = d_{\tilde{G}}(b_j, b_{j+1})$ holds for $0 \leq j < h$ by referring to the case when $h=1$, we can get that $d_G(b_s, b_t) = d_{\tilde{G}}(b_s, b_t)$.

So we prove that $d_G(b_s, b_t) = d_{\tilde{G}}(b_s, b_t)$ holds for all scenarios and boundary strategies. \square

We collect and summarize the time complexity of PSP strategies (Pre-Boundary, No-Boundary, Post-Boundary) in terms of index construction, query processing, and index update in Table 4.

9.3 Conversion of Vertex-Cut Partitions

Though graph partition can be categorized into *edge-cut* and *vertex-cut*, all of our proposed techniques are illustrated in the form of *edge-cut* for clear presentation since the vertex-cut partitions can be converted to edge-cut partitions. Specifically, we generalize the *vertex-cut* to *edge-cut* by duplicating those vertices that are cut into different partitions and connecting the duplicated vertices with their original vertices through an edge of zero weight. Suppose in a graph G , a vertex x is cut into $l + 1$ different partitions $\{G_0, G_1, \dots, G_l\}$ in a *vertex-cut partition* with x and its neighbors $\{x_{n_i}\} (0 \leq i \leq l)$ belonging to subgraph G_i . To obtain its equivalent *edge-cut partition*, we transform G to G' : $\forall x \in X$ (*cut vertex set*), keep the connection between x and its partial neighbors $\{x_{n_0}\}$, duplicate x as x_i connecting neighbors x_{n_i} , and connect x with x_i by an edge with $e(x, x_i) = 0$. Then we partition G' by using *edge-cut partition*: cut each added edge (x, x_i) , which leads to Lemma 10.

LEMMA 10. *The edge-cut partition of G' by cutting those added edges (x, x_i) is equivalent to the vertex-cut partition of G by cutting the vertices in X .*

Following by, we verify that the partition equivalence has no effect on the shortest distance computation.

THEOREM 11. $\forall s, t \in V, d_G(s, t) = d_{G'}(s, t)$.

PROOF. We classify all the scenarios into two cases:

Case 1: $p_G(s, t)$ ($p_{s,t}$ for short) does not pass through any $x \in X$. It indicates that $p_{s,t}$ is totally within one partition $G_i (1 \leq i \leq k)$. Since the vertex-cut partition of G and the edge-cut partition of G' are equivalent, $d_G(s, t) = d_G(s, t) = d_{G'}(s, t)$ holds.

Case 2: $p_{s,t}$ pass through $x \in X$. We suppose that $p_{s,t} = \langle s, \dots, x_f, x, x_b, \dots, t \rangle$ with $x_f \in G_i, x_b \in G_j$, then it can correspond to $p'_{s,t} = \langle s, \dots, x_f, x_i, x, x_j, x_b, \dots, t \rangle$ in G' . Since $e(x_i, x) = e(x, x_j) = 0, l(p_{s,t}) = l(p'_{s,t})$ holds which prove that $d_G(s, t) = d_{G'}(s, t)$ is correct as well. \square

9.4 Details of Update-oriented N-CH-P Index

We take the N-CH-P index as an example to introduce our proposed PSP indexes. The N-CH-P adopts the *no-boundary* strategy for index construction, query processing, and index update. Besides, *CH* is chosen as the SP algorithm for both the overlay and partition indexes since it is fast to update and has much better query efficiency than *direct search*. In what follows, we will elaborate on the index construction, query processing, and index maintenance of N-CH-P, respectively.

Index Construction. Algorithm 1 demonstrates the N-CH-P index construction. In particular, we first leverage the planar partition method such as PUNCH [18] to obtain the partition result of G (Line 1). After that, we parallelly construct the partition indexes $\{L_i\}$ by the CHINDEXING function [60, 87] (Lines 2-4). The key idea of CHINDEXING function is to iteratively contract the least important vertex v to obtain the distance-preserving shortcuts (Lines 15-18). Step 2 builds the overlay graph according to the initial inter-partition edges and the contraction results of partition indexes. It is worth noting that the *no-boundary* strategy entails constructing an all-pair clique among the boundary vertices of B_i by querying on $\{L_i\}$ to build the overlay graph. However, this approach could be time-consuming if the number of boundaries is large as the CH query efficiency is low. The final step is to build the overlay index on \tilde{G} , which is also conducted by running the CHINDEXING function (Lines 10-11).

Algorithm 1: N-CH-P Index Construction

Input: Graph $G = \{V, E\}$
Output: N-CH-P index $L = \{\tilde{L}, \{L_i\}\}$

- 1 $\{G_i | 1 \leq i \leq k\} \leftarrow$ partition result of G by PUNCH [18]
- 2 // Step 1: Partition Index Construction
- 3 **parallel_for** $i \in [1, k]$
- 4 $L_i \leftarrow$ CHINDEXING(G_i);
- 5 // Step 2: Overlay Graph Construction
- 6 $\tilde{G} \leftarrow E_{inter}$; ▷ Get the initial inter-partition edges
- 7 **parallel_for** $i \in [1, k]$
- 8 **for** $v \in B_i$ and $\forall u \in L_i(v)$ **do**
- 9 $\tilde{G} \leftarrow \tilde{G} \cup L_i(v, u)$; // $L_i(v, u)$ is the shortcut among v and u
- 10 // Step 3: Overlay Index Construction
- 11 $\tilde{L} \leftarrow$ CHINDEXING(\tilde{G});
- 12 **return** $L = \{\tilde{L}, \{L_i\}\}$;
- 13 **Function** CHINDEXING(G):
- 14 $L(v) \leftarrow \phi, \forall v \in V$;
- 15 **for** $v \in V$ in increasing vertex order **do**
- 16 $L(v) \leftarrow v$'s adjacent edges in G ;
- 17 insert/update the all-pair clique among $N_G(v)$ to G ;
- 18 $V \leftarrow V \setminus v$; remove v 's adjacent edges from G ;
- 19 **return** L ;

Query processing. It is worth noting that the no-boundary strategy has the same process for cross-partition and same-partition

query processing (both rely on the distance concatenation on the overlay index and partition index). Nevertheless, we can prune the search space of the N-CH-P index by only searching on the overlay index or partial partition index. Algorithm 2 presents the pseudo-code of N-CH-P query processing. In particular, when both s and t are in the overlay graph, we only need to conduct the CH search on the overlay index \tilde{L} according to Theorem 1. The CH search is implemented in `QUERYCH` function (Lines 9-27). Specifically, for the forward search, we assign a min-priority queue PQ_f , a search termination flag F_f , a flag vector C_f indicating whether a vertex has been popped from PQ_f , a distance vector D_f storing the distance to the source vertex s , and initialize PQ_f with $\langle s, 0 \rangle$ and $D_f[s]$ with 0 (the same goes for backward search but with subscript b , Lines 10-12). When both PQ_f and PQ_b are non-empty, and the termination condition lines 14-16 are not met, we conduct forward and backward searches iteratively on the CH index L (Lines 13-??). If both of them do not belong to \tilde{G} , we conduct the CH search on the combination of \tilde{L} , L_p , and L_q , where L_p and L_q are the partition indexes involving s and t . Otherwise, in the case that only s (or t) belongs to \tilde{G} , we only conduct the CH search on \tilde{L} and L_p (or L_q). In short, the N-CH-P is essentially equivalent to the CH index [60, 87] in terms of query processing.

Algorithm 2: N-CH-P Query Processing

Input: Query $Q(s, t)$ with $s \in G_p, t \in G_q$, N-CH-P index $L = \{\tilde{L}, \{L_i\}\}$
Output: Shortest distance of $Q(s, t)$

```

1  $d \leftarrow \infty$ ;
2 if  $s \in \tilde{G}$  and  $t \in \tilde{G}$  then
3    $d \leftarrow \text{QUERYCH}(s, t, \tilde{L})$ ; ▷ Query on overlay index  $\tilde{L}$ 
4 else
5   if  $s \notin \tilde{G}$  and  $t \notin \tilde{G}$  then  $d \leftarrow \text{QUERYCH}(s, t, \tilde{L} \cup L_p \cup L_q)$ ;
6   if  $s \notin \tilde{G}$  and  $t \in \tilde{G}$  then  $d \leftarrow \text{QUERYCH}(s, t, \tilde{L} \cup L_p)$ ;
7   if  $s \in \tilde{G}$  and  $t \notin \tilde{G}$  then  $d \leftarrow \text{QUERYCH}(s, t, \tilde{L} \cup L_q)$ ;
8 return  $d$ ;
9 Function QUERYCH( $s, t, L$ ):
10 initialize min-priority queues  $PQ_f$  and  $PQ_b$  with  $\langle s, 0 \rangle$  and  $\langle t, 0 \rangle$ ;
11  $d \leftarrow \infty$ ;  $F_f, F_b \leftarrow \text{false}$ ;  $C_f[v], C_b[v] \leftarrow \text{false}, \forall v \in V$ ;
12  $D_f[v], D_b[v] \leftarrow \infty, \forall v \in V$ ;  $D_f[s] \leftarrow 0, D_b[t] \leftarrow 0$ ;
13 while  $PQ_f$  is not empty and  $PQ_b$  is not empty do
14   if  $F_f = \text{true}$  and  $F_b = \text{true}$  then break;
15   if  $F_f = \text{true}$  and  $PQ_b$  is empty then break;
16   if  $F_b = \text{true}$  and  $PQ_f$  is empty then break;
17   // Forward Search
18   if  $PQ_f$  is not empty and  $F_f = \text{false}$  then
19      $\langle v, dis \rangle \leftarrow PQ_f.\text{Top}()$ ;  $PQ_f.\text{Pop}()$ ;  $C_f[v] \leftarrow \text{true}$ ;
20     if  $dis > d$  then  $F_f \leftarrow \text{true}$ ;
21     if  $C_b[v] = \text{true}$  and  $dis + D_b[v] < d$  then
22        $d \leftarrow dis + D_b[v]$ ;
23     for  $u \in L(v)$  do
24       if  $C_f[u] = \text{false}$  and  $dis + L(v, u) < D_f[u]$  then
25          $D_f[u] \leftarrow dis + L(v, u)$ ;  $PQ_f.\text{Update}(u, D_f[u])$ ;
26   // Backward Search. Similar to the forward search, omit here.
27 return  $d$ ;
```

Index maintenance. Algorithm 3 presents the pseudo-code of N-CH-P index update. Given a batch of update U , we first classify it into inter-partition updates \tilde{U} , and in-partition updates U_1, \dots, U_k corresponding to the updates in G_1, \dots, G_k (Line 1). Similar to the index construction procedure, the N-CH-P index maintenance also has three main steps. The first step maintains the partition indexes

for all affected partitions in parallel (Lines 2-6). This is implemented by running `CHINDEXUPDATE` function [60]. For each affected partition, such as G_i , it outputs the updated shortcuts set u_i , which is further fed to step 2 for obtaining the final overlay graph updates \tilde{U} . Given \tilde{U} as input, step 3 also maintains \tilde{L} by `CHINDEXUPDATE` function.

Algorithm 3: N-CH-P Index Maintenance

Input: Batch update set U
Output: Updated N-CH-P index L

```

1  $\tilde{U}, \{U_1, \dots, U_k\} \leftarrow \text{UPDATECLASSIFY}(U)$ ; ▷ Classify updates by distribution
2 // Step 1: Partition Index Maintenance
3  $u_i \leftarrow \phi, \forall i \in [1, k]$ ; //  $u_i$  stores the the updated shortcuts for  $G_i$ 
4 parallel_for  $i \in [1, k]$  and  $U_i \neq \phi$ 
5    $u_i \leftarrow \text{CHINDEXUPDATE}(U_i, L_i)$ ; //  $u_i$  is the updated shortcut set
6 // Step 2: Get Overlay Graph Updates
7 parallel_for  $i \in [1, k]$  and  $u_i \neq \phi$ 
8   for  $e(u, v) \in u_i$  do
9     if  $e(u, v) \in \tilde{G}$  and  $|e(u, v)| \neq |e_{\tilde{G}}(u, v)|$  then
10        $\tilde{U} \leftarrow \tilde{U} \cup e(u, v)$ ;
11 // Step 3: Overlay Index Maintenance
12  $\text{CHINDEXUPDATE}(\tilde{U}, \tilde{L})$ ; ▷ Overlay index update
13 return  $L$ ;
```

9.5 Additional Experimental Results

Exp 7: Effect of the Bandwidth. We leverage CT to evaluate the effect of bandwidth. As shown in Figure 17, the indexing time first decreases and then increases with the increase of bandwidth on FL and GO. The index update time also has such a tendency on FL. In other cases, a smaller treewidth leads to a smaller index size and query time. This is because large treewidth can result in a dense core for CT, thus dramatically enlarging the pruning point records that are necessitated by the PLL update [93]. We set the default treewidth as 40 since it has satisfactory performance.

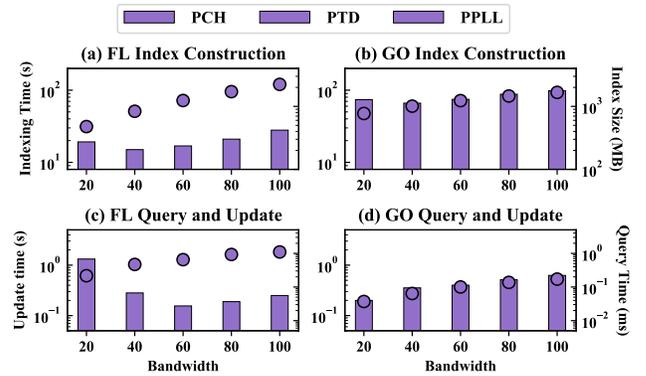


Figure 17: Effect of the Bandwidth.

Exp 8: Performance of Partition Methods When Varying k (Full Version). We report the performance of 8 classic partition methods by varying the partition number k from 4 to 256. As shown in Figure 18 (a)-(p), the overall border number $|B|$ generally increases while the average border number $|\overline{B}_i|$ decreases with the

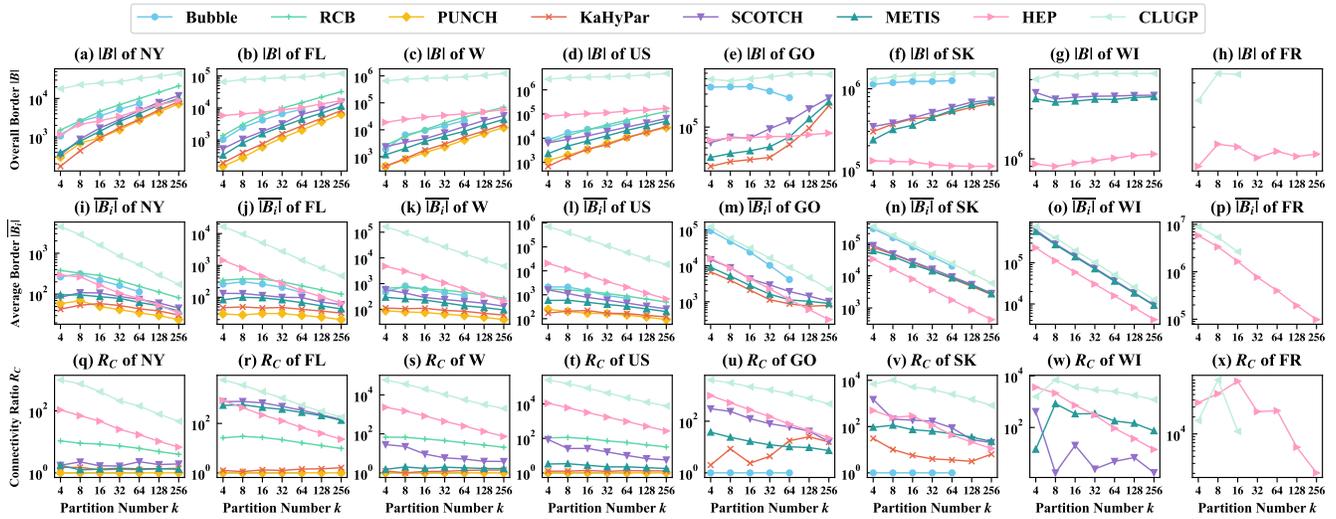


Figure 18: Performance of Partition Methods When Varying Partition Number k .

increase of k . This indicates that having a larger partition number can potentially speed up the query processing of the partition index due to a smaller average partition border number. However, it also leads to a larger overlay graph, which can slow down the query processing and construction of the overlay index. Therefore, there should be a trade-off between the overall border number and the average partition border number, and the best partition number typically varies for different PSP indexes. Furthermore, as per Figure 18 (q)-(x), among the partition methods used, only PUNCH and Bubble have a partition connectivity ratio R_C of 1. This implies

that the partitions generated by other partition methods generally contain multiple unconnected components. Since PTD requires the vertices within the same partition to be connected, only PUNCH and Bubble suit it. The graph shown in Figure 18 also provides insight into the optimal planar partition methods for different types of graphs. It reveals that PUNCH, KaHyPar, and METIS offer better performance for road networks than other methods. When dealing with complex networks, it is advisable to use KaHyPar, METIS, or HEP. In the case of the largest graph FR, only HEP and CLUGP are capable of completing graph partitioning within 24 hours.